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TRANSFORMATION GROUP C^*-ALGEBRAS WITH CONTINUOUS TRACE. II

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1. INTRODUCTION

In this note, G will be a locally compact group acting continuously on a locally compact Hausdorff space Ω. The translate of an $x \in \Omega$ by an $s \in G$ will be denoted by $s \cdot x$. The pair (G, Ω) will be referred to as a transformation group, and the familiar transformation group C^*-algebra associated with (G, Ω) [3] will be denoted by $C^*(G, \Omega)$. Of course, these objects are special cases of C^*-dynamical systems and C^*-crossed products, respectively. The question of when $C^*(G, \Omega)$ has continuous trace has attracted considerable attention lately and, in a sense, has been fairly well answered. Nevertheless, a number of related and more delicate questions remain. Our objective in this note is to contribute to one of these.

In [8], Green found necessary and sufficient conditions for $C^*(G, \Omega)$ to have continuous trace under the assumption that G is freely acting; i.e., under the assumption that $s \cdot x = x$ implies $s = e$. To be precise, [8, Theorem 17] asserts that if G is freely acting and if (G, Ω) is second countable, then $C^*(G, \Omega)$ has continuous trace if and only if compact subsets of Ω are wandering in the following sense: for each compact set $K \subseteq \Omega$, the set

$$\{s \in G : sK \cap K \neq \emptyset\}$$

is pre-compact in G. Green actually proved more than this. He showed that when G is freely acting and when compact subsets of Ω are wandering, then $C^*(G, \Omega)$ is isomorphic to the C^*-algebra determined by a continuous field of Hilbert spaces over the spectrum of $C^*(G, \Omega)$. To say the same thing differently, Green showed that when G is freely acting and when compact subsets of Ω are wandering, then $C^*(G, \Omega)$ has continuous trace, and furthermore, the Dixmier-Douady invariant of $C^*(G, \Omega)$ vanishes (cf. [2, 10.9.3]).

In [14], the second author determined when $C^*(G, \Omega)$ has continuous trace in a variety of cases without assuming that G is freely acting. His results are somewhat less definitive than Green's and are more complicated to state in their complete generality. Moreover, when $C^*(G, \Omega)$ has continuous trace the methods of [14] contain no information about the Dixmier-Douady invariant. One might suspect quite reasonably that for a general transformation group, the problem of deciding when the Dixmier-Douady invariant vanishes is a formidable one. The reason, of course, is that at the very least the problem depends upon how the isotropy or stability groups[1] vary and upon the Dixmier-Douady invariant of the C^*-algebra of each of the stability groups. Our objective in this note, then, is to investigate the Dixmier-Douady invariant of $C^*(G, \Omega)$ under the hypothesis that G is abelian. In this case, the group C^*-algebras of the stability groups are abelian, and thus, the Dixmier-Douady invariant always vanishes. On the other hand, the stability groups may still vary in a quite complicated fashion, and so our investigation is not without some interest.

Our method of attack is to associate an auxilliary groupoid $\mathcal{G}$ with (G, Ω) and to utilize the theory of groupoid C^*-algebras developed in [11]. We shall see that there is a canonical two-cocycle Δ on $\mathcal{G}$ with values in $\mathbf{T}$ that depends only on (G, Ω). In our main theorem, Theorem 2.3, we show that under certain hypotheses, $C^*(G, \Omega)$ is isomorphic to $C^*(\mathcal{G}, \Delta)$. (These hypotheses guarantee that $C^*(G, \Omega)$ has continuous trace and when (G, Ω) is second countable, they are also necessary for $C^*(G, \Omega)$ to have continuous trace.) We shall show too, under these hypotheses, that if Δ is trivial, then $C^*(G, \Omega)$ is strongly Morita equivalent to $C_0(\Lambda)$, where Λ is the spectrum of $C^*(G, \Omega)$. This, in turn, shows that the Dixmier-Douady invariant vanishes (cf. [10]). We do not know if there is a converse; i.e., we do not know if Δ must vanish when $C^*(G, \Omega)$ has continuous trace and the Dixmier-Douady invariant vanishes. In fact, we do not know if the intervention of Δ is entirely gratuitous. That is, we do not know of any example when $C^*(G, \Omega)$ has continuous trace and Δ is not trivial. Nevertheless, our result leads to a number of pleasing corollaries which do not seem to be accessible by other means.

For example, in Lemma 3.2, we show that when there is a continuous cross section to the canonical map from Ω to the space of orbits, Ω/G, then Δ is trivial. As another example, we show in Proposition 5.2 that Δ is trivial whenever there is a "continuous section to the quotients groups of $\hat{G}$ which varies continuously with the stability groups". Since such a section always exists when the action is free, our results, together with those in [14], provide a new proof of Theorem 14 in [8] when the group G is abelian.

In the future, we plan to investigate more fully the relation between the groupoid cocycle Δ and the Dixmier-Douady invariant for $C^*(G, \Omega)$. It is our opti-

[1] Recall that the stability group of a point $x \in \Omega$ is $\{s \in G : s \cdot x = x\}$.

mistic belief that they are intimately related. In any event, we believe that determining whatever relation there may be is the most piquant problem remaining in the subject.

2. PRELIMINARIES

Let (G, Ω) be a locally compact transformation group. We shall make the following assumptions throughout:

(2.1) G is abelian.

(2.2) All compact subsets of Ω are G-wandering in the sense of Definition 2.4 in [14]. That is, if $G \times \Omega/\sim$ is the quotient topological space obtained by identifying (s, x) with (t, y) if and only if $x = y$ and $s \cdot x = t \cdot x$, then we assume that, for any compact set $K \subseteq \Omega$, the set $\{(s, x) \in G \times \Omega/\sim \, : \, x \in K \text{ and } sK \cap K \neq \emptyset\}$ is relatively compact in $G \times \Omega/\sim$.

(2.3) The stability groups vary continuously on Ω [14, Definition 2.1].

These conditions imply that $C^*(G, \Omega)$ has continuous trace [14, Theorem 2.7]. Moreover, if (G, Ω) is second countable and G is abelian then (2.2) and (2.3) are necessary conditions for $C^*(G, \Omega)$ to have continuous trace [14, Theorem 5.1].

We let Σ denote the space of closed subgroups of G with the Fell topology [4]. For $K \in G$, let α_K denote a left Haar measure on K such that $\{\alpha_K\}$ are a continuous choice of measures on Σ. Recall that such a choice of measures has the property that the map

$$K \mapsto \int_K f(t)\, \mathrm{d}\alpha_K(t)$$

is continuous on Σ for every $f \in C_c(G)$ [6, p. 908]. For convenience α_{S_x} will be denoted by α_x, where S_x is the stability group at x.

Similarly, we let $\hat{\Sigma}$ denote the space of closed subgroups of $\hat{G}$. Note that the map $x \mapsto S_x^{\perp} (=\{\sigma \in \hat{G} : \sigma(t) = 1 \text{ if } t \in S_x\})$ is continuous from Ω to $\hat{\Sigma}$ in view of (2.3) and [16]. We fix a continuous choice of measures on $\hat{\Sigma}$, $\{\beta_{H^{\perp}}\}_{H \in \Sigma}$, and we denote by β_x the measure on $S_x^{\perp}$.

Finally, we let μ_x be the Haar measure on G/S_x normalized so that for $f \in C_c(G)$

$$\int_G f(s)\, \mathrm{d}\alpha_G(s) = \int_{G/S_x} \int_{S_x} f(st)\, \mathrm{d}\alpha_x(t)\, \mathrm{d}\mu_x(s).$$

We remark that the β_x may be chosen so that β_x is the measure required in the Fourier inversion formula for functions of positive type in $L^1(G/S_x, \mu_x)$; however, we shall not need this fact.

DEFINITION 2.1. Let A denote the quotient topological space obtained from $\Omega/G\times\hat{G}$ by identifying $(G\cdot x,\ \rho)$ and $(G\cdot y, \chi)$ if $G\cdot x = G\cdot y$ and $\bar{\rho}\chi\in S_x^\perp$.

It follows from our assumptions, [14, Proposition 2.17], and [15, Theorem 5.3], that A is homeomorphic to $C^*(G, \Omega)^\wedge$. In particular, A is locally compact Hausdorff. The last statement may also be deduced from [14, Lemma 2.3].

To prove our results, we shall want to view $C^*(G, \Omega)$ in a slightly non-standard way. In particular, we require several quotient spaces.

DEFINITION 2.2. Let $\mathscr{G} = G\times\Omega\times\hat{G}/\sim$ be the quotient space obtained from $G\times\Omega\times\hat{G}$ by identifying (s, x, ρ) with (r, y, σ) if $x = y$, $r^{-1}s\in S_x$, and $\bar{\rho}\sigma\in S_x^\perp$. The space $G\times\Omega/\sim$ was identified in assumption 2.2 and its companion, $\Omega\times\hat{G}/\sim$, is defined similarly to be the quotient of $\Omega\times\hat{G}$ obtained by identifying (x, σ) with (y, τ) if and only if $x = y$ and $\bar{\sigma}\tau\in S_x^\perp$.

With our assumptions, and using Lemma 2.3 of [14], these spaces can be shown to be locally compact, Hausdorff, and each quotient map is open. (Note that G may be replaced by $\hat{G}$ in the proof of Lemma 2.3 in [14] without any other changes.) We shall blur the distinction between points in $G\times\Omega\times\hat{G}$, $G\times\Omega$, and $\Omega\times\hat{G}$ and their images in the quotient spaces. This should cause no confusion in context.

Observe that $\mathscr{G}$ may be given the structure of a locally compact groupoid. Two elements, (s, x, ρ) and (r, y, σ), are composable if and only if $\rho = \sigma$ and $y = s^{-1}\cdot x$. In that case,

$$(s, x, \rho)\,(r, s^{-1}\cdot x, \rho) = (sr, x, \rho).$$

The inverse of (s, x, ρ) is $(s^{-1}, s^{-1}x, \rho)$. The unit space is easily identified with $\Omega\times\hat{G}/\sim$, and we have a natural Haar system defined as follows: if $u = (e, x, \rho)$ then for $F\in C_c(\mathscr{G})$

$$\lambda^u(F) = \int_{G/S_x} F(s,\ x, \rho)\,\mathrm{d}\mu_x(s).$$

Property (i) of [11, Definition 1.2.2] is obvious, and property (iii) follows from the fact that G/S_x is an abelian group. Property (ii) is more subtle, but follows from [14, Proposition 2.18] as follows. Using this proposition, it is possible to find $b\in C_c^+(\Omega, G)$ such that

$$F(s, x, \rho) = F(s, x, \rho)\int_{S_x} b(x, st)\,\mathrm{d}\alpha_x(t),$$

and consequently we find that

$$\lambda^{(x,\rho)}(F) = \int_G F(s, x, \rho)b(x, s)\,\mathrm{d}\alpha_G(s).$$

Since the integrand has compact support on $G\times(\Omega\times\hat{G}/\sim)$, continuity in (x, ρ) is clear.

Recall that if Δ is a continuous 2-cocycle in $Z^2(\mathscr{G}, \mathbf{T})$ then $C_c(\mathscr{G})$ admits a $*$-algebraic structure defined by

$$f*g(s, x, \rho) = \int_{G/S_x} f(r, x, \rho)g(r^{-1}s, r^{-1}x, \rho)\Delta((r, x, \rho), (r^{-1}s, r^{-1}x, \rho))\, d\mu_x(r)$$

$$f^*(s, x, \rho) = \overline{f(s^{-1}, s^{-1}\cdot x, \rho)}\, \Delta((s, x, \rho), (s^{-1}, s^{-1}\cdot x, \rho)).$$

The completion of $C_c(\mathscr{G})$ in the norm $\|\cdot\|_I$ defined by

$$\|f\|_I = \max\left\{\sup_{(x,\rho)} \int_{G/S_x} |f(s, x, \rho)|\, d\mu_x(s), \sup_{(x,\rho)} \int_{G/S_x} |f(s^{-1}, s^{-1}\cdot x, \rho)|\, d\mu_x(s)\right\}$$

is a Banach $*$-algebra denoted $L^I(\mathscr{G}, \Delta)$. The enveloping C^*-algebra of $L^I(\mathscr{G}, \Delta)$ will be denoted $C^*(\mathscr{G}, \Delta)$. Recall, too, [11, p. 50] that the isomorphism class of $C^*(\mathscr{G}, \Delta)$ depends only on the class of Δ in $H^2(\mathscr{G}, \mathbf{T})$. In particular, when Δ represents the trivial element, $C^*(\mathscr{G}, \Delta) \simeq C^*(\mathscr{G}, 1)$, which we normally write as simply $C^*(\mathscr{G})$.

In the last section we shall show that there is a continuous function δ on $G\times \Omega/\sim)\times \hat{G}$ such that

(2.4) δ has modulus one, and

(2.5) $\delta(s, x, \rho\sigma) = \sigma(s)\delta(s, x, \rho)$ if $\sigma \in S_x^\perp$.

Using δ, we can then define a 2-cocycle $\Delta = \Delta_{(G,\Omega)}$, on $\mathscr{G}^2$ as follows: let $\xi = (s, x, \rho)$ and $\eta = (r, s^{-1}\cdot x, \rho)$ and define

$$\Delta(\xi, \eta) = \delta(s, x, \rho)\delta(r, s^{-1}\cdot x, \rho)\,\overline{\delta(sr, x, \rho)}.$$

Of course, Δ is independent of choice of representatives of ξ and η in $(G\times\Omega/\sim)\times\hat{G}$ in view of property (2.5) above.

Our main theorem can now be stated.

THEOREM 2.3. *Suppose that (G, Ω) satisfies our basic assumptions,* 2.1, 2.2 *and* 2.3. *Then $C^*(G, \Omega)$ is isomorphic to $C^*(\mathscr{G}, \Delta)$ and $C^*(\mathscr{G})$ is strongly Morita equivalent to $C_0(\Lambda)$. So, in particular, when Δ is trivial, $C^*(G, \Omega)$ is isomorphic to the C^*-algebra defined by a continuous field of Hilbert spaces.*

We shall take up the proof of Theorem 2.3 in the following sections.

COROLLARY 2.4. *In the same situation as the theorem, suppose that (G, Ω) is second countable, that $\Delta = \Delta_{(G,\Omega)}$ is trivial, that each stability group has infinite index in G, and that Λ is a finite dimensional topological space. Then $C^*(G, \Omega)$ is iso-*

morphic to $C_0(\Lambda) \otimes \mathscr{K}$, *where* $\mathscr{K}$ *is the algebra of compact operators on some infinite dimensional Hilbert space.*

Proof. It follows from [14, Theorem 5.1], [14, Proposition 2.17], and [15, Lemma 4.15] that every irreducible representation of $C^*(G, \Omega)$ can be realized on $L^2(G/S_x, \mu_x)$ for some x. Since $L^2(G/S_x, \mu_x)$ is infinite dimensional for each x, $C^*(G, \Omega)$ is locally trivial (i.e. locally isomorphic to $C_0(Y) \otimes \mathscr{K}$) [2, 10.9.5]. In particular, $C^*(G, \Omega)$ is stable (i.e. $C^*(G, \Omega)$ is isomorphic to $C^*(G, \Omega) \otimes \mathscr{K}$) [9, Proposition 1.12]. However, our main theorem and [1, Theorem 1.2] imply $C^*(G, \Omega) \otimes \mathscr{K}$ is isomorphic to $C_0(\Lambda) \otimes \mathscr{K}$. QED

Let $f \in C_c(G, \Omega)$ and consider the following function on $\mathscr{G}$.

$$\varkappa(f)(s, x, \rho) = \overline{\delta(s, x, \rho)} \int_{S_x} f(st, x)\, \rho(t)\, \mathrm{d}\alpha_x(t). \tag{2.6}$$

LEMMA 2.5. (i) $\varkappa(f)$ *is continuous on* $\mathscr{G}$.

(ii) $\varkappa(f)(s, x, \rho\sigma) = \overline{\sigma(s)}\varkappa(f)(s, x, \rho)$ *for* $\sigma \in S_x^\perp$.

(iii) *There is a compact subset,* K, *of* $G \times \Omega/\sim$ *such that* $\varkappa(f)(s, x, \rho) = 0$ *if* $(s, x) \notin K$.

(iv) *For every* $\varepsilon > 0$, *there is a compact subset of* $\hat{G}$, K_ε, *such that for every* $(s, x) \in G \times \Omega/\sim$,

$$\{\rho \in \hat{G} : |\varkappa(f)(s, x, \rho)| \geqslant \varepsilon\} \subseteq K_\varepsilon S_x^\perp.$$

The proof of the above lemma requires a "uniform" version of the Riemann-Lebesgue Lemma. If $f \in C_c(G)$ and $\rho \in \hat{G}$, then define

$$\hat{f}(H, \rho) = \int_H f(t)\, \rho(t)\, \mathrm{d}\alpha_H(t).$$

The fact we need is

LEMMA 2.6. *For all* $\varepsilon > 0$, *there is a compact subset,* K_ε, *of* G *such that, if* $H \in \Sigma$, *then*

$$\{\rho \in \hat{G} : |\hat{f}(H, \rho)| \geqslant \varepsilon\} \subseteq K_\varepsilon H^\perp.$$

Proof of Lemma 2.5. Parts (ii) and (iii) are immediate. Part (i) follows from [14, Lemma 2.5].

To verify (iv), let $f \in C_c(G, \Omega)$ with $\operatorname{supp} f \subseteq K \times C$. Notice that

$$t \to F(st, x)$$

is in $C_c(S_x)$ with support in $s^{-1}K \cap S_x$. Thus, if $g \in C_c^+(G \times \Omega)$ with g identically one on $K \times C$, then

$$\int_{S_x} g(st, x)\, \mathrm{d}\alpha_x(t)$$

is continuous with compact support on $G\times\Omega/\sim$. In particular, $\alpha_x(s^{-1}K\cap S_x)$ is bounded for all $(s,x)\in G\times\Omega$. It follows that if f_α converges to f in the inductive limit topology on $C_c(G\times\Omega)$, then $\varkappa(f_\alpha)$ converges uniformly to $\varkappa(f)$ on $G\times\Omega/\sim$.

On the other hand, sums of the form $\sum_i \varphi_i\cdot\psi_i$, with $\varphi_i\in C_c(G)$ and $\psi_i\in C_c(\Omega)$ are dense in the inductive limit topology on $C_c(G,\Omega)$. Thus, the result follows from Lemma 2.6 which implies $\varkappa(\varphi\cdot\psi)$ satisfies (iv) for every $\varphi\in C_c(G)$ and $\psi\in C_c(\Omega)$.

QED

Proof of Lemma 2.6. Let $\mathscr{S}=\{(H,t)\in\Sigma\times G : t\in H\}$. Note that $\mathscr{S}$ is locally compact and Hausdorff. One can define a $*$-algebraic structure on $C_c(\mathscr{S})$ by

$$f*g(H,t)=\int_H f(H,v)\,g(H,v^{-1}t)\,\mathrm{d}\alpha_H(v),$$

and

$$f^*(H,t)=\overline{f(H,t^{-1})}.$$

Notice that $C_c(\mathscr{S})$ is a dense subalgebra of Fell's subgroup C^*-algebra [cf. 5], $C^*(\mathscr{S})$. Let Δ be the quotient topological space obtained from $\Sigma\times\hat{G}$ by identifying (H,ρ) and (H,χ) if $\bar{\rho}\chi\in H^\perp$. We claim $\Delta_{\mathscr{S}}$ is homeomorphic to the maximal ideal space of $C^*(\mathscr{S})$. In fact, $F\in C_c(\mathscr{S})$ defines a function on $\Delta_{\mathscr{S}}$ by

$$\hat{F}([H,\rho])=\int_H F(H,t)\overline{\rho(t)}\,\mathrm{d}\alpha_H(t).$$

First we show this suffices to prove the lemma. Any $f\in C_c(G)$ defines an element of $C_c(\mathscr{S})$ by

$$F(H,t)=f(t).$$

Moreover, $\hat{F}([H,\rho])=\hat{f}(H,\rho)$, and must vanish at infinity on $\Delta_{\mathscr{S}}$. Since the natural map of $\Sigma\times\hat{G}$ onto $\Delta_{\mathscr{S}}$ is open [14, Lemma 2.3], we are done.

To establish our claim we must introduce still another algebra. Let $A_0=C_c(\Sigma\times G)$ be viewed as a dense subalgebra of $A=C(\Sigma)\otimes C^*(G)\simeq C_0(\Sigma\times\hat{G})$. Of course, if $F\in A_0$, then the corresponding function on $\Sigma\times\hat{G}$ is defined by

$$\hat{F}(H,\rho)=\int_G F(H,s)\overline{\rho(s)}\,\mathrm{d}\alpha_G(s).$$

It is easy to see that the natural map of $\Sigma\times\hat{G}$ into $\operatorname{Prim}C^*(\mathscr{S})$, defined by $(H,\rho)\to$ $\to\operatorname{Ker}L_{(H,\rho|H)}$, is continuous, where $L_{(H,\rho|H)}$ denotes the obvious irreducible representation of $C^*(\mathscr{S})$. Moreover, this map clearly factors through the quotient, $\Delta_{\mathscr{S}}$. Thus all we need show is that it is open.

Let $\mathscr{I}(A)$ denote the space of closed, two-sided ideals of A equipped with Fell's inner hull-kernel topology, cf. [15, discussion following Proposition 2.2]. It follows from the continuity of the inducing operation [5, Theorem 4.2] that the map from $\operatorname{Prim} C^*(\mathscr{S})$ to $\mathscr{I}(A)$ defined by

$$\operatorname{Ker} L_{(H,\rho|H)} \mapsto \operatorname{Ker} M_{(H,\operatorname{Ind}_H^G(\rho|H))}$$

is continuous, where $M_{(H,\operatorname{Ind}_H^G(\rho|H))}$ is defined by sending $F \in A_0$ to

$$\int_G F(H,s) \operatorname{Ind}_H^G(\rho|H)(s)\, \mathrm{d}\alpha_G(s).$$

Now suppose that $\operatorname{Ker}(L_{(H_\alpha,\rho_\alpha|H)})$ converges to $\operatorname{Ker}(L_{(H,\rho|H)})$ in $\operatorname{Prim} C^*(\mathscr{S})$. If we identify ideals of $C_0(\Sigma \times \hat{G})$ with closed subsets of $\Sigma \times \hat{G}$ in the usual way, then $\operatorname{Ker}(M_{(H,\operatorname{Ind}_H^G(\rho|H)})$ corresponds to $(H, \rho H^\perp)$. Thus, it follows from [15, Lemma 2.4] that there is a subnet, $\{(H_\beta, \rho_\beta)\}$ and $\sigma_\beta \in H_\beta^\perp$ such that $(H_\beta, \rho_\beta\sigma_\beta)$ converges to (H, ρ) in $\Sigma \times \hat{G}$. This shows that the natural map of $\Sigma \times \hat{G}$ onto $\operatorname{Prim} C^*(\mathscr{S})$ is open as promised. QED

Finally, let $\mathscr{P} = \{(x, y) \in \Omega \times \Omega : G \cdot x = G \cdot y\}$. $\mathscr{P}$ is easily seen to be closed in $\Omega \times \Omega$; hence, $\mathscr{P}$ is locally compact, Hausdorff. We may think of $\mathscr{P}$ as the principal part of the transformation group (G, Ω), viewed as a groupoid.

LEMMA 2.7. *Given assumptions* (2.2) *and* (2.3), *the map, λ, from $G \times \Omega$ to $\mathscr{P}$ defined by $(s, x) \mapsto (x, s \cdot x)$ is open and defines a homeomorphism, $\tilde{\lambda}$, of $G \times \Omega/\sim$ with $\mathscr{P}$.*

Proof. Note that λ factors through $G \times \Omega/\sim$. Thus, it suffices to show that λ is open as a map on $G \times \Omega/\sim$ ([14, Lemma 2.3]). One can deduce this from general results on groupoids [11, p. 17], but here is a self-contained proof.

Suppose $(x_\alpha, s_\alpha \cdot x_\alpha)$ converges to (x, y) in $\mathscr{P}$. It will suffice to show that there is a subnet so that (s_β, x_β) converges to a (s, x) in $G \times \Omega$.

Let U and V be compact neighborhoods of x and y respectively, and set $T = U \cup V$. Thus we eventually have

$$(s_\alpha, x_\alpha) \in \{(r, y) : y \in T \text{ and } sT \cap T \neq \emptyset\}.$$

Our conclusion follows since the right hand side is relatively compact in $G \times \Omega/\sim$ by assumption (2.3). QED

3. PROOF OF THEOREM 2.3

Let $C_\infty(\mathscr{G}, \Delta)$ denote the set of functions on $\mathscr{G}$ which satisfy conditions (i) — (iv) of Lemma 2.5. Of course, $C_\infty(\mathscr{G}, \Delta)$ may be viewed as a *-algebra containing $C_c(\mathscr{G}, \Delta)$. We are interested in the following representations of $C_\infty(\mathscr{G}, \Delta)$. For each

$x \in \Omega$ and $\rho \in \hat{G}$, let R_x^ρ be defined on $L^2(G/S_x, \mu_x)$ by

$$R_x^\rho(f)\varphi(s) = \int_{G/S_x} \delta(r, s\cdot x, \rho) f(r, s\cdot x, \rho)\, \varphi(r^{-1}\cdot s)\, \mathrm{d}\mu_x(r),$$

where $f \in C_\infty(\mathscr{G}, \Delta)$ and $\varphi \in L^2(G/S_x)$. We then define a pre-C^*-norm on $C_\infty(\mathscr{G}, \Delta)$ by

$$\|f\|_r = \sup_{\substack{x\in\Omega \\ \rho\in G}} \|R_x^\rho(f)\|,$$

and denote the completion of $C_\infty(\mathscr{G}, \Delta)$ by $C_r^*(\mathscr{G}, \Delta)$.

Of course, R_x^ρ can easily be seen to be equivalent to the regular representation on $\delta_{(x,\rho)}$ of $C^*(\mathscr{G}, \Delta)$ (cf. [11, Definition 2.1.8]), where $\delta_{(x,\rho)}$ is the point mass at (x, ρ). $C_r^*(\mathscr{G}, \Delta)$ is the reduced groupoid C^*-algebra ([11, Definition 2.2.8]).

Although it is not difficult to show that $\mathscr{G}$ is measurewise amenable [11, Definition 2.3.6], and hence that $C_r^*(\mathscr{G}, \Delta)$ is isomorphic to $C^*(\mathscr{G}, \Delta)$ [11, Proposition 2.3.2], we are able to establish this isomorphism directly in our special circumstances without much effort.

We shall devote the remainder of the section to showing that the proof of Theorem 2.3 can be reduced to the following two results.

PROPOSITION 3.1. *Assuming* (2.1), (2.2), *and* (2.3) *are valid, then* $C_r^*(\mathscr{G})$ *is (strongly) Morita equivalent to* $C_0(\Lambda)$. *In particular, the representation of* $C_r^*(\mathscr{G})$ *induced from point evaluation at (the class of)* (x, ρ) *in* Λ *is equivalent to* R_x^ρ.

LEMMA 3.2. *Suppose* (G, Ω) *is a locally compact transformation group satisfying* (2.1), (2.2), *and* (2.3). *If in addition there is a continuous cross section for the orbit space (i.e. for the map* $\Omega \to \Omega/G$*), then* $\Delta_{(G,\Omega)}$ *is trivial in* $H^2(\mathscr{G}, \mathbf{T})$.

Proof of Theorem 2.3. Note that the map $\varkappa$ defined in equation (2.6) defines a $*$-homomorphism of $C_c(G, \Omega)$ into $C_\infty(\mathscr{G}, \Delta)$. A straightforward computation shows that, for $F \in C_c(G, \Omega)$,

$$R_x(\varkappa(F)) = L_x^\rho(F),$$

where L_x^ρ is the irreducible representation of $C^*(G, \Omega)$ defined in [15, Lemma 4.14]. In particular, it follows from [14, Proposition 2.17 (iii)] and the above, that $\varkappa$ defines an injection of $C^*(G, \Omega)$ into $C_r^*(\mathscr{G}, \Delta)$.

On the other hand, the wandering hypothesis implies Ω/G is locally compact, Hausdorff. In particular, there is a natural map, $R = R_{\Omega/G}$, of $C_0(\Omega/G)$ into the center of $M(C_r^*(\mathscr{G}, \Delta))$ defined by

$$R(\varphi)\cdot f(s, x, \rho) = \varphi(G\cdot x) f(s, x, \rho).$$

Thus, if L is any irreducible representation of $C_r^*(\mathcal{G}, \Delta)$, then the canonical extension of L to $M(C_r^*(\mathcal{G}, \Delta))$ defines (by restriction) a non-trivial complex homomorphism of $C_0(\Omega/G)$. In fact, there is a $x_0 \in \Omega$ such that for each $\varphi \in C_0(\Omega/G)$ and $f \in C_c(\mathcal{G}, \Delta)$,

$$L(R(\varphi)f) = \varphi(G \cdot x_0)L(f).$$

Using the fact that the representations R_x determine the norm on $C_r^*(\mathcal{G}, \Delta)$ and the above, we see that the kernel of L contains the ideal in $C_r^*(\mathcal{G}, \Delta)$ generated by those functions in $C_c^*(\mathcal{G}, \Delta)$ which vanish on all $(s, x, \rho) \in \mathcal{G}$ with $x \in G \cdot x_0$. In fact, L factors through $C_r^*(\mathcal{G}_0, \Delta_0)$, where $(\mathcal{G}_0, \Delta_0)$ is the groupoid and cocycle associated to the transitive transformation group $(G, G \cdot x_0)$. Since the orbit space is a single point, we may apply Lemma 3.2 and Proposition 3.1 to conclude that $C_r^*(\mathcal{G}_0, \Delta_0)$ is Morita equivalent to $C_0(\rho \hat{S}_{x_0})$. It follows that L must be equivalent to a R_x for some $(x, \rho) \in \Omega \times \hat{G}$, and that L is a CCR representation.

In summary, $C_r^*(\mathcal{G}, \Delta)$ is liminal. Moreover, it is not difficult, using the above, to see that $\varkappa(C^*(G, \Omega))$ is a "rich subalgebra" of $C_r^*(\mathcal{G}, \Delta)$ in the sense of [2, 11.1.1]. Therefore $C^*(G, \Omega)$ and $C_r^*(\mathcal{G}, \Delta)$ are isomorphic by [2, 11.1.4].

Recall that $C^*(G, \Omega)$ may also be viewed as the groupoid C^*-algebra of the transformation group groupoid $\mathcal{G}_1 = (G, \Omega)$ [11, Example 1.2.5 a)]. It is easy to see that $\varkappa$ is L^I-norm decreasing as a map of $L^I(\mathcal{G}_1)$ into $C_\infty(\mathcal{G}) \subseteq L^I(\mathcal{G}, \Delta)$. In particular, L^I-norm decreasing representations of $L^I(\mathcal{G}, \Delta)$ define L^I-norm decreasing representations of $L^I(\mathcal{G}_1)$ which are of course C^*-norm decreasing representations. Since $C^*(G, \Omega)$ is isomorphic to $C_r(\mathcal{G}, \Delta)$, the original representation must be a $\|\cdot\|_r$-norm decreasing representation of $C_\infty(\mathcal{G})$. Thus, $C_r^*(\mathcal{G}, \Delta)$ coincides with $C^*(\mathcal{G}, \Delta)$.

Thus, we have shown that $C^*(G, \Omega)$ is isomorphic to $C^*(\mathcal{G}, \Delta) = C_r^*(\mathcal{G}, \Delta)$. When Δ is trivial, Proposition 3.1 shows that $C^*(G, \Omega)$ is strongly Morita equivalent to $C_0(\Lambda)$; in this case, $C^*(G, \Omega)$ is determined by a continuous field of Hilbert spaces (cf. [10], Proposition C1). QED

4. PROOF OF PROPOSITION 3.1

We shall proceed by identifying $C_r^*(\mathcal{G})$ with the imprimitivity algebra of a $C_0(\Lambda)$-rigged space; the proposition then follows immediately [12, Section 6].

Our module is $C_c(\Omega \times \hat{G}/\sim)$ and it will be denoted by X. For convenience let $C_0(\Lambda)$ be denoted by B. We make the following definitions for u and v in X, f in $C_c(\mathcal{G})$, and b in B.

$$(4.1) \qquad f \cdot u(x, \rho) = \int_{G/S_x} f(r, x, \rho)\, u(r^{-1} \cdot x, \rho)\, d\mu_x(r)$$

$$\langle u, v\rangle_{\mathscr{G}}(s, x, \rho) = u(x, \rho)\,\overline{v(s^{-1}\cdot x, \rho)} \tag{4.2}$$

$$u\cdot b(x, \rho) = u(x, \rho)\, b(G\cdot x, \rho) \tag{4.3}$$

$$\langle u, v\rangle_B(G\cdot x, \rho) = \int_{G/S_x} \overline{u(r^{-1}\cdot x, \rho)}v(r^{-1}\cdot x, \rho)\, \mathrm{d}\mu_x(r). \tag{4.4}$$

The next set of formulae are verified with straightforward calculations.

$$\langle f\cdot u, v\rangle_{\mathscr{G}} = f * \langle u, v\rangle_{\mathscr{G}}. \tag{4.5}$$

$$\langle u, v\cdot b\rangle_B = \langle u, v\rangle_B\, b. \tag{4.6}$$

$$u\cdot\langle v, w\rangle_B = \langle u, v\rangle_{\mathscr{G}}\cdot w. \tag{4.7}$$

$$\overline{\langle v, w\rangle}_B = \langle w, v\rangle_B. \tag{4.8}$$

$$\langle v, v\rangle_B \geqslant 0. \tag{4.9}$$

PROPOSITION 4.1. *X is a B-rigged space.*

Proof. It is evident from equations (4.6), (4.8), and (4.9) that we need only establish that $\mathrm{Span}\{\langle v, v\rangle_B : v\in X\}$ is dense in $C_0(\Lambda)$. Thus, it will suffice to notice that the natural map of $C_c(\Omega\times\hat{G}/\sim)$ to $C_c(\Lambda)$, defined by sending f to

$$\int_{G/S_x} f(r^{-1}\cdot x, \rho)\, \mathrm{d}\mu_x(r)$$

is onto. This is a consequence of the wandering hypothesis and [14, Lemma 2.18(i)]. QED

Recall that the imprimitivity algebra of X, $\mathscr{E}^B$, is that generated by the operators on X of the form

$$T_{(u,v)}(w) = u\langle v, w\rangle_B.$$

It now is evident from (4.7) that $\mathscr{E}^B$ may be identified with a subalgebra (of the completion in the appropriate norm) of $C_c(\mathscr{G})$. Furthermore, we shall show that the $\mathscr{E}^B$-norm agrees with the $\|\cdot\|_r$-norm, and hence $\mathscr{E}^B$ may be viewed as a subalgebra of $C_r^*(\mathscr{G})$. Towards this end, note that irreducible representations of B are just the point evaluations. Let $(G\cdot x, \rho)\in\Lambda$ be fixed, and consider the representation of $\mathscr{E}^B$, N_x^ρ, induced from evaluation at $(G\cdot x, \rho)$. Of course, N_x^ρ acts on the completion of X with respect to the inner product

$$\langle u, v\rangle = \langle u, v\rangle_B(G\cdot x, \rho),$$

and

$$N_x^\rho(f)u = f\cdot u.$$

Moreover, the map of X into $L^2(G/S_x, \mu_x)$ defined by

$$\Psi(u)(s) = u(s\cdot x, \rho)$$

is an isometry with respect to $\langle\cdot,\cdot\rangle$. It is a straightforward matter to check that Ψ defines a unitary which implements and equivalence between N_x^ρ and the representation R_x^ρ (when restricted to the appropriate subalgebra of $C_c(\mathscr{G})$) defined in the previous section; our claim about norms now follows.

We may complete the proof of Proposition 3.1 by showing that $\langle\cdot,\cdot\rangle_{\mathscr{G}}$ spans a dense subset of $C_c(\mathscr{G})$. It will suffice to make several observations.

First, the span of $\langle\cdot,\cdot\rangle_{\mathscr{G}}$ is an algebra (an ideal in fact). Secondly, it follows from Lemma 2.7 that the wandering hypothesis implies that the map from $\mathscr{G}$ to $\mathscr{P}\times\hat{G}/\sim$ defined by $(s, x, \rho)\to(x, s^{-1}\cdot x, \rho)$ is a homeomorphism. In particular, since functions of the form

$$f(x, y, \rho) = \sum_{i=1}^{n} u_i(x, \rho)v_i(y, \rho)$$

with $u_i, v_i\in X$ are dense in the inductive limit topology on $C_c(\mathscr{P}\times\hat{G}/\sim)$, it is evident that functions of the form

$$f_*(s, x, \rho) = \sum_{i=1}^{n} u_i(x, \rho)v_i(s^{-1}\cdot x, \rho)$$

are dense in $C_c(\mathscr{G})$; this completes the proof.

We now turn to the existence of δ and the proof of Lemma 3.2.

5. PROOF OF LEMMA 3.2 AND FURTHER EXAMPLES

In this section we show that Δ is always defined, and exhibit several cases where Δ must trivialize. Of course, the existence of Δ follows from the next lemma.

LEMMA 5.1. *There always exists a continuous function* δ *of modulus one on* $(G\times\Omega/\sim)\times\hat{G}$ *such that, for all* $s\in G, x\in\Omega$, $\rho\in\hat{G}$, *and* $\sigma\in S_x^\perp$, $\delta(s, x, \rho\sigma) = \sigma(s)\delta(s, x, \rho)$.

Proof. Let $G\times\Sigma/\sim$ be the quotient of $G\times\Sigma$ obtained by identifying (s, H) with (t, K) if and only if $H = K$ and $sH = tH$; let $\Sigma\times\hat{G}/\sim$ be the quotient of $\Sigma\times\hat{G}$ obtained by identifying (H, σ) with (K, τ) if and only if $H = K$ and $H^\perp\sigma = H^\perp\tau$; finally, let $G\times\Sigma\times\hat{G}/\sim$ be the quotient of $G\times\Sigma\times\hat{G}$ obtained by identifying

(s, H, σ) with (t, K, τ) if and only if $H = K$, $sH = tH$, and $H^\perp\sigma = H^\perp\tau$. As in Definition 2.2 and the following discussion, it is not hard to show that these spaces are locally compact and Hausdorff. Moreover, it will clearly suffice to produce a δ' of modulus one on $(G\times\Sigma/\sim)\times\hat{G}$ such that

$$\delta'(s, H, \rho\sigma) = \sigma(s)\delta'(s, H, \rho),$$

for each $\sigma\in H^\perp$. Indeed, given δ', we put

$$\delta(t, x, \rho) = \delta'(t, S_x, \rho).$$

Our reason for replacing Ω by Σ in the above is to take advantage of the facts that $G\times\Sigma/\sim$ and $\Sigma\times\hat{G}/\sim$ are paracompact, as we shall show. Unfortunately, these assertions do not appear to follow from general theory as the open image of a paracompact space need not be paracompact (however, we know of no example for which this fails when the image is locally compact, Hausdorff). Fortunately, we may take advantage of our special situation. Of course, it will suffice to prove our assertion only for $G\times\Sigma/\sim$ since Σ and $\hat{\Sigma}$ are homeomorphic [16].

Let G_1 be a σ-compact open subgroup of G. Also, let $\Gamma\subseteq G$ be a set of representatives of cosets of G/G_1. The family $\{\alpha G_1\times\Sigma\}_{\alpha\in\Gamma}$ is a disjoint clopen cover of $G\times\Sigma$ consisting of σ-compact sets. Let U_α be the saturation of $\alpha G_1\times\Sigma$ in $G\times\Sigma$:

$$U_\alpha = \{(\alpha sh, H) : s\in G_1, h\in H\}.$$

Since the natural map of $G\times\Sigma$ onto $G\times\Sigma/\sim$ is open, the U_α are open as well as σ-compact. In fact, if Γ_0 is any subset of Γ, then

$$V = \bigcup_{\alpha\in\Gamma_0} U_\alpha$$

is open; we claim V is also closed. Towards this end, let $(x_0, H_0)\in G\times\Sigma\setminus V$, and let π_0 be the natural map of G onto G/H_0. Since $\pi_0(G_1)$ is an open subgroup of G/H_0, it follows that G/H_0 is the disjoint union of clopen cosets of $\pi_0(G_1)$. In particular, the intersection, V_{H_0}, of V with $\{(s, H_0): s\in G\}$ is closed. On the other hand $\Gamma_0\subseteq V_{H_0}$. It follows that there is a function, $g\in C_c(G)$, such that

$$\tilde{g}(s, H) = \int_H g(st)\, d\alpha_H(t)$$

is one at (x_0, H_0) and zero on all (s, H_0) with $s\in\Gamma_0$. Note that $\tilde{g}\in C_c(G\times\Sigma/\sim)$ [14, Lemma 2.5 and proof of Proposition 2.18], and hence, is uniformly continuous in the sense that for all $\varepsilon > 0$, there are neighborhoods $\mathcal{N}$ of e and $\mathcal{M}$ of H_0 such that if $r^{-1}s\in\mathcal{N}$ and $H\in\mathcal{M}$, then

$$|\tilde{g}(s, H) - \tilde{g}(r, H)| < \varepsilon.$$

In particular, there is a neighborhood of H_0, $\mathcal{M}$, such that

$$\tilde{g}(x_0, H) > 1/2, \quad \text{and} \quad \tilde{g}(s, H) < 1/2$$

if $H \in \mathcal{M}$ and $s \in \Gamma_0$. This implies that $\tilde{g}(s, H) < 1/2$ on $\mathcal{M} \times \Sigma \cap V$ and that the complement of V is open.

We may assume that Γ is well-ordered. Thus, we can define

$$U'_\alpha = U_\alpha \setminus \bigcup_{\lambda < \alpha} U_\lambda .$$

From the above, the $\{U'_\alpha\}$ form a clopen, σ-compact cover of $G \times \Sigma/\sim$; our assertion about paracompactness follows.

Returning to the proof of the lemma, let T be a transversal (which may fail to be measurable) for the natural map of $\Sigma \times \hat{G}$ onto $\Sigma \times \hat{G}/\sim$. We claim that it will suffice to produce, given any compact subset of K of G, a continuous function, $u = u_K$, whose real part is strictly positive on $K \times T$, and which satisfies (5.1). Then, since each U'_α is clopen and σ-compact, we can construct a function u_α whose real part is strictly positive on $U'_\alpha \times T$ and such that u_α is zero off U'_α. Thus,

$$u = \sum_\alpha u_\alpha$$

is well defined and nowhere zero on $\Sigma \times \hat{G}$; the desired function is obtained by dividing by the modulus.

Let $b = (H_0, \rho_0) \in T$ be fixed for the moment. We easily find $F_b \in C_c(\Sigma \times \hat{G})$ such that

$$\left| \int_H F_b(H_0, \rho_0\sigma)\, \sigma(s)\, \mathrm{d}\beta_H(\sigma) - 1 \right| < 1/2$$

for all $s \in K$. It follows from [14, Lemma 2.5] that

$$u_b(s, H, \rho) = \int_{H^\perp} F_b(H, \rho\sigma)\sigma(s)\, \mathrm{d}\beta_H(\sigma)$$

is continuous; hence, there are neighborhoods, U_b of H_0 and V_b of ρ_0, such that

$$\mathrm{Re}\,(u_b(s, H, \rho)) > 0,$$

for $s \in K$, $H \in U_b$, and $\rho \in V_b$. In fact, by cutting u_b down by an appropriate function in $C_c(G \times \Omega \times \hat{G}/\sim)$, we may assume u_b has support in

$$A = \{(s, H, \rho\sigma) : s \in K,\ (H, \rho) \in U_b \times V_b, \text{ and } \sigma \in S_x^\perp\}.$$

Fortunately, since T is a transversal, $A \cap K \times T$ and $(K \times U_b \times V_b) \cap K \times T$ coincide. In particular, we may assume without loss of generality that

$$\mathrm{Re}(u_b) \geqslant 0 \quad \text{on } K \times T,$$

and

$$\mathrm{Re}(u_b) > 0 \quad \text{on } K \times U_b \times V_b.$$

Now let W_b denote the saturation of $U_b \times V_b$ in $\Sigma \times \hat{G}/\sim$. Of course, the collection $\{W_b\}_{b \in T}$ covers $\Omega \times \hat{G}/\sim$. Since $\Sigma \times \hat{G}/\sim$ is paracompact, we may find a locally finite refinement [13, § 1] of $\{W_b\}_{b \in T}$. Thus, by cutting the u_b down where necessary, we may assume that we have a cover, $\{W_{b_\alpha}\}_{\alpha \in I}$, such that, for any $(H, \rho) \in \Sigma \times \hat{G}$, only finitely many b_α are such that $u_{b_\alpha}(s, H, \rho) \neq 0$. Therefore we may define

$$u_K(s, H, \rho) = \sum_{\alpha \in I} u_{b_\alpha}(s, H, \rho). \qquad \text{QED}$$

Proof of Lemma 3.2. Let δ be as in Lemma 5.1. Furthermore, let γ be the continuous cross section of Ω/G to Ω (which we view as a function from Ω to Ω). Let θ, mapping $\mathscr{P}$ to $G \times \Omega/\sim$, be the inverse to $\bar{\lambda}$ in Lemma 2.7. We only need to define

$$\tilde{\delta}(s, x, \rho) = \delta(\theta(\gamma(x), x), x, \rho)\overline{\delta(\theta(\gamma(x), s^{-1}\cdot x), s^{-1}\cdot x, \rho)}.$$

Since the $\tilde{\Delta}$ defined by $\tilde{\delta}$ is identically one, it follows that Δ always represents the trivial element in $H^2(\mathscr{G}, \mathbf{T})$. QED

PROPOSITION 5.3. *If there is a continuous cross section,* λ, *from* $\Omega \times \hat{G}/\sim$ *to* $\Omega \times \hat{G}$, *then* $\Delta_{(G,\Omega)}$ *is trivial.*

Proof. Let π_2 denote the projection of $\Omega \times \hat{G}$ onto its second factor. We define

$$\varkappa(x, \rho) = \overline{\pi_2(\lambda(x, \rho))} \cdot \rho.$$

Notice that $\varkappa(x, \rho) \in S_x^\perp$, and that

$$\varkappa(x, \rho\sigma) = \varkappa(x, \rho)\sigma$$

for all $x \in \Omega$, $\rho \in \hat{G}$, and $\sigma \in S_x^\perp$. In particular we may define

$$\delta(s, x, \rho) = \varkappa(x, \rho)(s).$$

This δ yields a trivial Δ. QED

We remark that if the action is free, then there always is a cross section, namely $\lambda(x, \rho) \equiv 1$, the trivial character. Thus, when G is abelian, Green's result [8, Theorem 14] is a consequence of Theorem 2.3 and the above proposition.

It is also interesting to note that we may apply Theorem 2.3 to Example 5.4 in [14]. Since there is an obvious continuous cross section, we conclude that the algebra constructed there not only has continuous trace, but is defined by a continuous field of Hilbert spaces.

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