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COCYCLES ON THE CIRCLE

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1. INTRODUCTION

$\mathbf{T}$ is the circle group, and Γ a countable dense subgroup of $\mathbf{T}$. $\mathbf{T}$ will be written additively, so that its elements are real numbers added modulo 2π. A *cocycle* on $\Gamma \times \mathbf{T}$ is a measurable function from this product to a group, which satisfies a certain identity. We are interested in *multiplicative* cocycles, taking values in $\mathbf{T}$ and satisfying

$$A(\lambda + \tau, x) = A(\lambda, x)A(\tau, x + \lambda) \tag{1.1}$$

almost everywhere on $\mathbf{T}$ for all λ, τ in Γ; and in *additive* cocycles, with values in the real number system $\mathbf{R}$ and satisfying

$$v(\lambda + \tau, x) = v(\lambda, x) + v(\tau, x + \lambda) \tag{1.2}$$

almost everywhere for each λ, τ.

If v is an additive cocycle, obviously $\exp iv$ is a multiplicative cocycle.

A *coboundary* is a cocycle of the form

$$A(\lambda, x) = q(x + \lambda)/q(x), \quad v(\lambda, x) = w(x + \lambda) - w(x) \tag{1.3}$$

respectively, a.e. on $\mathbf{T}$ for each λ, where q is a measurable function on $\mathbf{T}$ of modulus 1 and w is a measurable real function on $\mathbf{T}$. Two cocycles are *cohomologous* if their ratio, or difference, respectively, is a coboundary.

If v is an additive coboundary, then obviously $\exp iv$ is a multiplicative coboundary.

A cocycle of either kind is called *trivial* if it is in the smallest group containing all coboundaries and all cocycles that are constant in x for each λ. Two cocycles are called *equivalent* if their ratio, or their difference, is trivial.

The translation group T in $L^2(\mathbf{T})$ is defined by

$$T_\lambda f(x) = f(x + \lambda) \quad (\lambda \text{ in } \Gamma). \tag{1.4}$$

(T_λ) is a unitary representation of Γ. Given a multiplicative cocycle A, a new representation is obtained by setting

$$S_\lambda f(x) = A(\lambda, x) f(x + \lambda). \tag{1.5}$$

The spectral theorem represents (S_λ) as the Fourier transform of a spectral measure on the compact abelian group K dual to Γ. Our objective is to know more about spectral measures that are obtained from cocycles in this way. We shall see that their spectral properties are connected with problems of Diophantine approximation.

Some proofs of statements below have been omitted because they are simple, or like proofs of analogous results in [2].

2. THE COMMUTATION RELATION

Denote by χ the function on $\mathbf{T}$ whose values are $\exp ix$, and by M the operation of multiplication by χ. Then it is easy to see that

$$S_\lambda M = e^{i\lambda} M S_\lambda \quad (\lambda \text{ in } \Gamma). \tag{2.1}$$

This is the *Weyl commutation relation* for these groups. In particular, the translation group T (corresponding to the cocycle whose values are 1) satisfies the relation.

THEOREM 1. *A representation (S_λ) of Γ satisfies the commutation relation if and only if it is obtained from a cocycle by* (1.5).

The easy proof is omitted. From this point, (S_λ) will always denote a representation of Γ satisfying the commutation relation.

THEOREM 2. *Only the trivial subspaces of $L^2(\mathbf{T})$ are invariant under the groups (M^n) and (S_λ).*

Let f be any element of $L^2(\mathbf{T})$, and g an element of the space that is orthogona to the smallest subspace containing f and invariant under all the unitary operators:

$$\int A(\lambda, x) f(x + \lambda) \bar{g}(x)\, e^{nix}\, dx = 0 \tag{2.2}$$

for all integers n and all λ in Γ. (Integrals are from 0 to 2π.) The relation for all n implies that the integrand vanishes a.e. for each λ. Since A is a unitary function,

$$f(x + \lambda) g(x) = 0 \text{ a.e.} \quad (\text{all } \lambda \text{ in } \Gamma). \tag{2.3}$$

From this it follows that one of f and g must vanish identically, and the theorem is proved.

THEOREM 3. *The spectral measure P of a representation (S_λ) is either purely Lebesgue, purely singular-continuous, or purely discrete with respect to Haar measure σ on K.*

Let H_j $(j = 1, 2, 3)$ be the subspaces of $L^2(\mathrm{T})$ in which P is, respectively, absolutely continuous, singular-continuous, and discrete. These subspaces are invariant under the projections of P and hence also under all S_λ; if we show that they are invariant under M and M^{-1} then they must be trivial, and this will prove that P has spectrum of pure type.

Let f belong to H_j. The commutation relation gives

$$(S_\lambda Mf, Mf) = e^{i\lambda}(S_\lambda f, f). \tag{2.4}$$

Now $(S_\lambda f, f)$ is a function on Γ that is the Fourier-Stieltjes transform of a measure on K of pure type j. The measure is merely translated when its transform is multiplied by $\exp i\lambda$, and remains of the same pure type. Thus Mf belongs to the same subspace H_j as f. It follows that also $M^{-1}f$ is in that subspace, and this completes the proof that P has spectrum of pure type.

Finally it is asserted that if the spectrum is absolutely continuous, then it is Lebesgue. The commutation relation implies that the null sets of P are invariant under translations from Γ. It is easy to see that every P-null set must have Lebesgue measure 0.

THEOREM 4. *The cocycle A is trivial if and only if P is of discrete type.*

Let $\exp ir$ be an eigenvalue of the group S, with eigenfunction q:

$$A(\lambda, x)q(x + \lambda) = e^{ir\lambda}q(x). \tag{2.5}$$

The translation group is ergodic; it follows that $|q|$ is constant, and can be taken equal to 1 a.e. . Now $\exp ir\lambda$ is a cocycle, constant in x for each λ, and thus (2.5) shows that A is trivial.

If A is a coboundary, then $S_\lambda = \bar{q}T_\lambda q$ (an equality of operators), so that S is unitarily equivalent to T. Since T has discrete spectrum the same is true of S. Finally, a cocycle that is constant in x for each λ is a character of Γ; multiplying A by this character merely translates the spectral measure, so that every trivial cocycle also leads to discrete spectrum.

THEOREM 5. *The multiplicity of P is uniform.*

The argument, using the commutation relation, is familiar.

3. EXTENSIONS OF COCYCLES

Let Γ be a countable dense subgroup of **T**, and Γ' another such group containing Γ. Given a cocycle A on Γ, what kind of extensions can A have to Γ'? Surprisingly, in some circumstances extensions are essentially unique if they exist at all.

THEOREM 6. *Every extension of a singular cocycle is singular; every restriction of a Lebesgue cocycle is Lebesgue. Both extensions and restrictions of trivial cocycles are trivial.*

Suppose that A is a cocycle on Γ' of Lebesgue type. There is a non-null function f in $L^2(\mathbf{T})$ such that $(S_\lambda f, f)$ is square-summable over Γ'. The restriction of this inner product to Γ is square-summable *a fortiori*. This shows that the absolutely continuous part of the spectrum of S, restricted to Γ, is non-trivial, and therefore S on Γ is also of Lebesgue type.

It is obvious that the restriction of a Γ'-coboundary to Γ is a coboundary (with the same cobounding function). It follows that the restriction of a trivial cocycle is trivial. Hence every extension of a singular cocycle is singular (for it is not discrete or Lebesgue).

The last assertion to be proved is that every extension of a trivial cocycle is trivial. A coboundary has an extension that is a coboundary; thus a trivial cocycle has an extension that is trivial. Dividing cocycles on Γ', we have to prove this proposition: if A is a cocycle on Γ' such that $A(\lambda, x) = 1$ for all λ in Γ, then A is trivial on Γ'.

For any λ and τ the cocycle identity shows that

$$A(\lambda, x)A(\tau, x + \lambda) = A(\tau, x)A(\lambda, x + \tau) \quad \text{a.e.} \tag{3.1}$$

Hence for λ in Γ we have

$$A(\tau, x + \lambda) = A(\tau, x) \text{ a.e.} \tag{3.2}$$

That is, the unitary function A_τ is invariant under translations from Γ. Since Γ is dense, A_τ is constant. Thus A is a trivial cocycle, and the theorem is proved.

COROLLARY. *A cocycle on Γ is trivial if its restriction to any dense subgroup of Γ is trivial.*

THEOREM 7. *Suppose that Γ is a subgroup of Γ' with index r, and that A is a cocycle of Lebesgue type on Γ' with multiplicity n. Then the restriction of A to Γ has multiplicity rn. (Either or both of r, n may be infinite.)*

The hypotheses imply that we can find n functions f_j in $L^2(\mathbf{T})$ such that the functions $(S_\lambda f_j)$, for all j and all λ in Γ', constitute an orthonormal basis for $L^2(\mathbf{T})$. Let (λ_k) be r elements of Γ' that are in distinct cosets of Γ. For each j and k

set $g_{jk} = S_{\lambda_k} f_j$. Then $(S_\lambda g_{jk})$, for λ in Γ, is an orthonormal Γ-cycle; as j and k vary we see that rn of these orthogonal cycles span $L^2(\mathbf{T})$.

The theorem does not obviously hold when A has singular spectrum, but it is easy to see at any rate that restriction *cannot decrease* multiplicity.

COROLLARY. *If A has simple Lebesgue spectrum, then every proper restriction of A has multiple Lebesgue spectrum.*

It is also obvious that A cannot have an extension with Lebesgue spectrum; but we shall get a better result.

COROLLARY. *For each group Γ there is a cocycle with Lebesgue spectrum of infinite multiplicity.*

Let Γ' be a countable group containing Γ with infinite index. A remarkable result of Mathew and Nadkarni [4] asserts that Γ' has a cocycle with Lebesgue spectrum. Our theorem now says that its restriction to Γ has infinite Lebesgue spectrum.

THEOREM 8. *A cocycle with simple Lebesgue spectrum has no proper extension.*

The spectral theorem, with our assumption of simple Lebesgue spectrum, implies that the group S acting in $L^2(\mathbf{T})$ is isomorphic to a group S' acting in $L^2(K)$ (the Lebesgue space on K based on Haar measure σ), where S'_λ is multiplication by the character λ of K. The operator M in $L^2(\mathbf{T})$ becomes a new operator M', which satisfies the commutation relation still with S'. We must determine the form of M'.

Let e be the character of Γ that maps each element on itself. With Γ written additively, this means $e(\lambda) = \exp i\lambda$. Define translation for functions on K: $Tf(x) = f(x - e)$. Then T satisfies the commutation relation with S':

$$S'_\lambda T = e^{i\lambda} T S'_\lambda \quad (\lambda \text{ in } \Gamma). \tag{3.3}$$

If q is any unitary function on K, qT also satisfies the relation, and these are the only operators that do. Therefore $M' = qT$ for some unitary function q.

We are to prove that S', a unitary representation of Γ in $L^2(K)$ satisfying the commutation relation with M', has no extension to a unitary representation of a larger group Γ' that still satisfies the commutation relation with M'. If such an extension exists, let τ be an element of Γ' not in Γ, and S'_τ the corresponding operator. Then S'_τ commutes with all S_λ (λ in Γ), and therefore is multiplication by a unitary function p. The commutation relation gives

$$pqT = e^{i\tau} qTp. \tag{3.4}$$

Interpreted, this equality of operators means that

$$p(x)q(x) = e^{i\tau} q(x) p(x - e). \tag{3.5}$$

The factors q cancel, leaving

$$p(x) = e^{i\tau}p(x - e). \tag{3.6}$$

In other words, the exponential factor is a coboundary for the action of T on K.

p has a Fourier series:

$$p(x) \sim \sum a(\lambda)\lambda(x) \quad (\lambda \text{ in } \Gamma). \tag{3.7}$$

From (3.6) we obtain

$$a(\lambda) = e^{i(\tau-\lambda)}a(\lambda) \quad (\lambda \text{ in } \Gamma). \tag{3.8}$$

Hence $a(\lambda) = 0$ unless $\lambda = \tau$. Now some coefficient $a(\lambda)$ is not 0 because p is not null; hence τ belongs to Γ. This contradicts our choice of τ, and shows that the representation of Γ could not be extended.

THEOREM 9. *Suppose that S is a Lebesgue representation of Γ, and that Γ' contains Γ with finite index. If S has an extension to Γ', then the extension has Lebesgue spectrum also.*

We omit the proof.

4. DIOPHANTINE APPROXIMATION

After these general theorems, we specialize to the group Γ in $\mathbf{T}$ generated by a single element α of infinite order. Define $Tf(x) = f(x + \alpha)$. For q a unitary function, let $S = qT$. The powers of S determine a multiplicative cocycle, but mainly we deal with q itself. We say that q is a coboundary if $q = Tp/p$ for some unitary function p; this is necessary and sufficient for the cocycle generated by q to be a coboundary in the full sense.

THEOREM 10. *Let p be an inner function. Then $S = pT$ has Lebesgue spectrum. If p is a finite Blaschke product then S has multiplicity equal to the number of zeros in p; otherwise its multiplicity is infinite.*

This result is due to Bagchi, Mathew and Nadkarni [1].

In particular, if p is a Blaschke factor with a single zero, then S has simple Lebesgue spectrum. It is an interesting question whether all these Blaschke factors are cohomologous. For some α at least they are.

THEOREM 11. *The Blaschke factor*

$$(\chi - s)/(1 - \bar{s}\chi) \quad (|s| < 1) \tag{4.1}$$

is cohomologous to χ if

$$\sum_{1}^{\infty} \left| \frac{s^n}{n(1 - \exp ni\alpha)} \right|^2 < \infty. \tag{4.2}$$

Dividing by χ, and replacing s by its conjugate, we want to show that the function

$$(1 - \bar{s}\chi)/(1 - s\chi) = \exp(-2i) \arg(1 - s\chi) \tag{4.3}$$

is a coboundary. If we show that

$$-\log(1 - s\chi(x)) = w(x + \alpha) - w(x) \tag{4.4}$$

for some function w, then by taking the imaginary part we see that $-2\arg(1 - s\chi)$ is an additive coboundary, and the assertion is proved.

The left side of (4.4) is a power series in χ. We shall find w in $H^2(\mathbf{T})$ with Fourier coefficients (a_n). By (4.4) these coefficients should satisfy

$$s^n/n = a_n(e^{ni\alpha} - 1) \quad (n > 0). \tag{4.5}$$

The sequence (a_n) defined by this formula is square-summable by (4.2), the function w with these coefficients satisfies (4.4), and the theorem is proved.

The criterion (4.2) is satisfied for all s, $0 < |s| < 1$, for certain α (with $\alpha/2\pi$ irrational), and for no such s for other α. The failure of (4.2) does not obviously imply that the conclusion of the theorem is wrong; we do not know the truth in general.

Nevertheless one naïve possibility can be disposed of easily: not *all* cocycles with simple Lebesgue spectrum are equivalent. For χ and $\bar{\chi}$ both have simple Lebesgue spectrum; but they are not equivalent, because their quotient, χ^2, is not trivial (on the contrary it has Lebesgue spectrum).

Next we want to discuss an interesting theorem of K. Petersen [6], related to a result of H. Kesten [3]. As before, α is a fixed real number such that $\alpha/2\pi$ is irrational; T is the translation by α in $L^2(\mathbf{T})$.

THEOREM 12. *Let β be any real number. In order to have*

$$\sum_{1}^{\infty} \left| \frac{\exp ni\beta - 1}{n(\exp ni\alpha - 1)} \right|^2 < \infty \tag{4.6}$$

it is necessary and sufficient that β have the form $2\pi k + n\alpha$ (k and n integers).

The fact that (4.6) holds if β has the given form is elementary and we omit the proof.

Before proving the opposite assertion, we make some remarks. Let $v(x) = x$ on $(0, 2\pi)$. Then v is not additively trivial; that is, there are no real measurable periodic function w and constant c such that

$$v(x) = w(x + \alpha) - w(x) + c \quad \text{a.e.} \tag{4.7}$$

For if there were, then $\chi(x) = \exp iv(x) = k \exp iw(x+\alpha)/\exp iw(x)$ would be multiplicatively trivial. But the obvious argument with Fourier series shows that no function q in $L^2(\mathbf{T})$ except 0 can satisfy

$$kq(x+\alpha) = e^{ix}q(x) \text{ a.e.} \tag{4.8}$$

Denote by D_β the difference operator: $D_\beta f(x) = f(x+\beta) - f(x)$. If f is an additive coboundary with cobounding function w, then $D_\beta f$ is a coboundary with cobounding function $D_\beta w$. The converse, however, is not obvious and undoubtedly often false. (See Theorem 13 below). However, for the given function v we have

LEMMA. *$D_\beta v$ is not an additive coboundary unless β has the special form of the theorem.*

This is the crucial point of Petersen's proof. $D_\beta v$ is constant on $(0, 2\pi)$ except for jumps of magnitude exactly 2π. Therefore $\exp iD_\beta v = c$, a constant. If $D_\beta v$ is an additive coboundary, then c is a multiplicative coboundary, which means that c is an eigenvalue of the translation operator T in $L^2(\mathbf{T})$. These eigenvalues are the numbers $\exp ni\alpha$, n an integer. Thus $v(x+\beta) - v(x) \equiv n\alpha \pmod{2\pi}$ a.e., for a certain fixed n. But the difference is $\beta \pmod{2\pi}$, so that $\beta \equiv n\alpha \pmod{2\pi}$, proving the lemma.

We finish the proof. Suppose that (4.6) holds. Except for a constant factor, the Fourier coefficients of $D_\beta v$ are $a_n = (\exp ni\beta - 1)/n$ $(n \neq 0)$. Set $b_n = a_n(\exp ni\alpha - 1)^{-1}$, a sequence that is square-summable by (4.6). If w is the function with Fourier coefficients (b_n), then $Tw - w = D_\beta v$, contradicting the lemma. Therefore (4.6) cannot hold except in the special cases mentioned.

The lemma has another proof based on Theorem 8. We know that $S = \chi T$ has simple Lebesgue spectrum in $L^2(\mathbf{T})$. (For an immediate direct proof, note that $(S^n 1)$ is a complete orthonormal system in $L^2(\mathbf{T})$.) Therefore the cocycle A on the group $(n\alpha)$ such that $A(\alpha, x) = \exp ix$ has no extension to any larger group. We derive a condition for A to have an extension to β with $A(\beta, x) = q$. Expanding $A(\alpha+\beta, x)$ in two ways gives

$$\chi(x)q(x+\alpha) = q(x)\chi(x+\beta). \tag{4.9}$$

Conversely, if this relation holds, then there is a cocycle A on the group generated by α and β with χ and q as its values at α, β respectively.

Thus (4.9) cannot hold for any unitary function q unless β belongs to the group generated by α. That is, $\chi(x+\beta)/\chi(x) = \exp iD_\beta v(x)$ is not a multiplicative coboundary, and so $D_\beta v$ is not an additive coboundary, except for such β.

This example is very simple; we only want to make the point that Theorem 8 provides a way to show that functions are not coboundaries.

THEOREM 13. *Let f be a real function such that $D_\beta f$ is a coboundary for each real β. Then f is additively trivial.*

This result is less elementary than might appear. Let $v(n, x)$ be the additive cocycle with $v(1, x) = f$. Then $\exp iD_\beta v$ is a multiplicative coboundary for each β: for each β

$$e^{iD_\beta v(n,x)} = q(\beta, x + n\alpha)/q(\beta, x) \text{ a.e.} \tag{4.10}$$

for same unitary function q that is measurable in x for each β. Our first objective is to show that q can be chosen measurable on the product space so that the equality holds for almost all pairs (β, x).

If we take the mean value in n, the numerator on the right has for limit the mean value of q_β, by the ergodic theorem. We call this limit $L(\beta)$. Since the left side is measurable in (β, x), $L(\beta)/q(\beta, x)$ is measurable. On the set where $L(\beta)$ is not 0 we replace q in (4.10) by $q(\beta, x)/L(\beta)$, so that equality still holds and the new function q is measurable in both variables. If we divide by its modulus, it is a unitary function.

On the set where $L = 0$, we have to modify this argument. Replace q by χq, introducing a factor $\exp in\alpha$ on the left side. Now the mean value is different from 0 on a new set, and it is easy to patch together a measurable unitary function to satisfy (4.10) on the set where at least one limit is not 0. Using the same argument with all powers of χ provides a function on the whole product space.

Now in (4.10) replace β by $\beta - x$, and then change β to y:

$$e^{i[v(n,y)-v(n,x)]} = q(y - x, x + n\alpha)/q(y - x, x). \tag{4.11}$$

On the right side, the numerator is the same as the denominator with both x and y increased by $n\alpha$. In other words, on the dynamical system $\mathbf{T}\times\mathbf{T}$ with the transformation

$$(x, y) \to (x + \alpha, y + \alpha), \tag{4.12}$$

the cocycle $\exp i[v(n, y) - v(n, x)]$ is a coboundary.

It follows that $v(n, x)$ is a trivial cocycle. (I believe that this fact was first proved by K. Schmidt, but never published.) The translation operator in $L^2(\mathbf{T}\times\mathbf{T})$ defined by (4.12) has discrete spectrum; it follows, as in Theorem 4, that coboundaries lead to unitary groups with discrete spectrum. Therefore the inner product

$$\iint e^{i[v(n,y)-v(n,x)]}\, q(x + n\alpha, y + n\alpha)\bar{g}(x, y)\, dx\, dy \tag{4.13}$$

is an almost periodic function of n, for every g in $L^2(\mathbf{T}\times\mathbf{T})$. Take for g a product $\bar{h}(x)h(y)$. For some h in $L^2(\mathbf{T})$ the double integral is not identically 0 in n, and it equals

$$\left|\int e^{iv(n,y)}h(y+n\alpha)\bar{h}(y)\,dy\right|^2. \tag{4.14}$$

If the operator $e^{iv}T$ in $L^2(\mathbf{T})$ had singular or Lebesgue spectrum, the mean value of (4.14) would be 0, which is not the case for a non-trivial almost periodic sequence. Thus $\exp iv(n,x)$ is trivial.

This argument applies to all real scalar multiples of f, with the result that $\exp itv(n,x)$ is multiplicatively trivial for all real t. Now a beautiful theorem of C. C. Moore and K. Schmidt [5] (originally proved using the result of Schmidt just above) asserts that v is additively trivial, which is what was to be proved.

5. QUESTIONS

Petersen's theorem should have an extension to more general sums. Let β and γ be distinct real numbers such that

$$\sum_1^\infty \left|\frac{(\exp ni\beta - 1)(\exp ni\gamma - 1)}{(\exp ni\alpha - 1)n}\right|^2 < \infty. \tag{5.1}$$

Can we conclude that one of β, γ at least has the form $2\pi k + n\alpha$?

Exactly as before, (5.1) implies that $D_\beta D_\gamma v$ is a coboundary, where $v(x) = x$ on $(0, 2\pi)$; we could hope that this is the case only if $D_\beta v$ or $D_\gamma v$ is a coboundary, which indeed implies (5.1). This would be true if we could prove the assertion: unless β has the special form of Theorem 12, the multiplicative cocycle based on $q = \exp iuD_\beta v$ has simple Lebesgue spectrum for some real u. We have no idea whether this is right.

Let α and β be independent in $\mathbf{T}$; and Γ the group they generate. We do not know whether any cocycle on Γ has simple Lebesgue spectrum. If we allow M to have infinite multiplicity, then it is easy to find such a cocycle. That is, we can find an operator-valued cocycle Q (in an infinite-dimensional Hilbert space) such that $(Q_\lambda T_\lambda)$ is a unitary representation of Γ in the space $L^2(\mathbf{T})$ of vector-valued functions, which has simple Lebesgue spectrum.

In this connection, let us mention that Theorem 8 remains true for operator--valued cocycles, because the multiplicity of M is not referred to in the proof.

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