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BOOK REVIEW:

The Cauchy Problem for Solutions of Elliptic Equations, **Nikolai N. Tarkhanov**, Mathematical Topics, vol. 7, Akademie Verlag, Berlin, 1995, 480 pag., ISBN 3-05-501663-7.

KEYWORDS: *Cauchy problem, system of pde with injective symbols, approximation of solutions.*

AMS SUBJECT CLASSIFICATION: 35J99, 35G10.

This is a provocative book in the sense that it attacks a subject which is of great interest, not yet perfectly understood, and in fact full of promises. The idea which generated this monograph is to what extent the Carleman formulas, (see Aizenberg's book: Carleman formulas in complex analysis; first applications, Kluwer, 1993) can be generalized, in the sense that the problem of analytic continuation to which these formulas were successfully used is just a very particular case of a Cauchy Problem for overdetermined systems of partial differential equations.

This is neither a textbook nor a reference one. But the author has succeeded in pointing to the general ideas and methods. Now, about the contents of this book. The main attention is centered about the overdetermined systems of p.d.e. with injective (or surjective symbols). After a very interesting introduction which summarizes, explains and puts in proper light the main results of the book, the first chapter is devoted to distribution theory. The accent is put on spaces of distributions defined in a neighborhood of compact sets and their duals; the spaces are so choosen that their duals can be identified with some subspaces of distributions defined on the same compact set. So one finds here a lot of results which are not standard (as spectral synthesis in Hölder spaces, etc.).

Chapter 2 deals with pseudodifferential operators in the spaces of distributions on closed sets. Here one also finds some nonstandard results, as boundedness results for Ψdo on manifolds with boundary.

The third chapter, "Capacity", introduces capacity associated with a semi-normed space of distributions. In the 70′ such generalized capacities were considered by a number of authors (Fuglede, Harvey-Polking, Maz'ya, etc.). The author gives a unified and systematic study of this generalized capacity in Hölder and Sobolev spaces; the accent is put on sets of zero capacity, which play an important role (as they are quite often removable sets of singularities).

Chapter 4 is devoted to generalization to the context of overdetermined systems of the notion of ellipticity. This leads to the known notion of system with injective (or dually) surjective symbols. Here a new approach is suggested by use of the concept of fundamental solution of an elliptic complex (in essence this is to be found in the author's book "Parametrix method in the theory of differential complexes, (in Russian), Nauka, Novosibirsk, 1990, and in an improved version, in english, to appear in Kluwer).

The next chapters (5–8) treat one of the main subjects of this book, namely approximation theorems for solutions of overdetermined p.d.e. systems with surjective symbols. These results have a long history behind, to mention only the deep results of Keldish devoted to the uniform approximation by harmonic functions. Here the capacity introduced before plays an essential role. The author gives sharp approximation results in spaces of smooth functions, Hölder spaces on compact sets, Sobolev spaces also on compact sets. This chapter is one of the main interests of the book.

Chapter 9 is dedicated to the study of generalized boundary values of solutions of systems with injective symbols (strangely enough, classical results as those of Lions-Magenes are not even cited). Anyhow, this technical chapter is a prerequisite to Chapter 10, where the Cauchy Problem for a system with injective symbol, which is ill-posed, of course, is studied. The author considers Sokhotskii-Plemejl formulas, and solvability condition which leads to an iff result of solvability which is more or less what one should have expected (i.e. the orthogonality condition which appears there). But what is more important is that the same ideas in the context of Hilbert spaces lead to constructive results, as the Hahn-Banach theorem can be replaced by the technique of Fourier series. So this is achieved in Chapter 11 (Method of Fischer-Riesz in the C.P. for a system with injective symbol, and Chapter 12 (Bases with double orthogonality in the C.P. for systems with injective symbols).

This review, however, cannot give a fair idea of the wealth of results of this book, the many interesting examples, and many suggestions for further work it contains. As it is natural, it is influenced by the Soviet School, and makes popular a lot of results that were previously scattered around the Soviet literature on the subject but that are all now subsummed to some general, natural ideas.

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