J. OPERATOR THEORY
36(1996), 147–156

HYPERINVARIANT SUBSPACES FOR CERTAIN COMPACT PERTURBATIONS OF AN OPERATOR

ISABELLE CHALENDAR

Communicated by Nikolai K. Nikolskii

ABSTRACT. Let A, B be linear operators acting in a Hilbert space, such that B or AB is in a von Neumann-Schatten class $\mathcal{C}_p$. Sufficient conditions on the geometry of the spectrum and on the growth of the resolvent are given for the existence of hyperinvariant subspaces of $A + B$.

KEYWORDS: *Operator, Hilbert space, hyperinvariant subspace, Growth of the resolvent, compact perturbation and Neumann-Schatten class.*

AMS SUBJECT CLASSIFICATION: Primary 47A10, 47A15; Secondary 30C20.

1. INTRODUCTION

Let $\mathcal{H}$ be a separable, infinite-dimensional complex Hilbert space. We denote by $\mathcal{L}(\mathcal{H})$ the algebra of all bounded linear operators acting on $\mathcal{H}$ and by $\mathcal{K}(\mathcal{H})$ the ideal of $\mathcal{L}(\mathcal{H})$ of all compact operators. Let T be a bounded linear operator of $\mathcal{H}$. A closed subspace $\mathcal{M}$ of $\mathcal{H}$ is said to be hyperinvariant for T if $S\mathcal{M} \subset \mathcal{M}$ for any operator S that commutes with T.

The purpose of the present note is to show the existence of hyperinvariant subspaces for operators of the form $T = A + B$ where A is an operator whose spectrum has an exposed arc (see below) and whose resolvent has a certain growth and where B or AB is a certain compact operator in a von Neumann-Schatten class $\mathcal{C}_p, p \geqslant 1$. The results of this note may be considered as a generalization of Theorem 3.5 and Theorem 3.8 of [1], Corollary 3.3 of [4]. The main improvement is the fact we deal with stronger growth of the resolvent.

2. PRELIMINARIES

Throughout this note, if $T \in \mathcal{L}(\mathcal{H})$, we denote by $\sigma(T), \sigma_p(T), \sigma_e(T)$ and $\rho(T)$ the spectrum, the point spectrum, the essential spectrum and the resolvent set respectively (see [2], Part I, VII 5). The set of all $\mu \in \sigma(T)$ for which $\mu\mathrm{Id}-T$ is one-to-one and for which the manifold $(\mu\mathrm{Id}-T)\mathcal{H}$ is dense in but not equal to $\mathcal{H}$ is called the continuous spectrum of T and is denoted by $\sigma_c(T)$. As usual the resolvent function of T is denoted by $R(\cdot\,;T)$. For any complex number λ and a subset E of the complex plane, we set $d(\lambda, E) = \inf\{|\lambda - z|, z \in E\}$. We understand a smooth arc to be such a one that has a continuous second derivative when parametrized with respect to arc length. We assume a Jordan curve J is positively oriented and for a fixed λ_0 on J, where J has a parametrization $\lambda = g(s)$ $(0 \leqslant s \leqslant l(J))$, in term of arc length s from λ_0, $g(0) = \lambda_0, g(s) = g(s + l(J))$, and $g(s)$ is continuous on J and $g'(s), g''(s)$ are continuous except points $\lambda_k = g(s_k), s_k < s_{k+1}, k = 1, 2, \ldots, n$ on J, where $l(J)$ denotes the whole length of J. We denote by J, a Jordan curve, which consists of a finite number of rectifiable smooth arcs in the complex plane.

DEFINITION 2.1. If A is a bounded linear operator, then $\sigma(A)$ contains an exposed arc J if there exists an open disk $\mathcal{D}_J$ such that $\mathcal{D}_J \cap \sigma(A)$ consists of a finite number of rectifiable smooth Jordan arcs in the complex plane.

The set of all operators $A \in \mathcal{L}(\mathcal{H})$ whose spectrum contains an exposed arc will be denoted by $\mathcal{L}(\mathcal{H}, A_e)$.

If $A \in \mathcal{L}(\mathcal{H}, A_e)$ and if J is an exposed arc of $\sigma(A)$, we make the following definition:

DEFINITION 2.2. We say that the resolvent of A has the growth condition (C) near J if:

$$\int_0^{\varepsilon} \log\log M(\delta)\, \mathrm{d}\delta < \infty$$

for some sufficiently small $\varepsilon > 0$, where $M(\delta)$ is defined by:

$$M(\delta) = \sup\{\|(\lambda\mathrm{Id} - A)^{-1}\|, d(\lambda, J) \geqslant \delta, \lambda \in \mathcal{D}_J\}.$$

Now we give a theorem due to [5].

THEOREM 2.3. *Let $T \in \mathcal{L}(\mathcal{H}, A_e)$ and let J (not reduced to a single point) be an exposed arc of $\sigma(T)$ such that the resolvent of T has the growth condition* (C) *near J. Then T has non-trivial hyperinvariant subspaces.*

Before giving our results, for the sake of convenience, we shall list some facts on the von Neumann-Schatten classes C_p, $p > 0$ established in [3], Part II, XI.9.

Let C be a compact operator on $\mathcal{H}$ and $|C| := (C^*C)^{\frac{1}{2}}$. Let $\lambda_1, \lambda_2, \ldots, \lambda_n, \ldots$ (resp. $\mu_1, \mu_2, \ldots, \mu_n, \ldots$) be the eigenvalues of C (resp. $|C|$), arranged in decreasing order and repeated according to multiplicity. We write $\lambda_n(C)$ (resp. $\mu_n(C)$) for the n-th eigenvalue of C (resp. $|C|$). We write $\|C\|_p := \left(\sum_n \mu_n(C)^p\right)^{\frac{1}{p}}, 0 < p < \infty$. The class $\mathcal{C}_p$ is the set of all compact operators C such that $\|C\|_p$ is finite.

THEOREM 2.4. *If* $C \in \mathcal{C}_p$, $p \geqslant 1$ *then:*

(i) $\left(\sum_n |\lambda_n(C)|^p\right)^{\frac{1}{p}} \leqslant \|C\|_p$.

(ii) *If* $A \in \mathcal{L}(\mathcal{H})$, $\|AC\|_p \leqslant \|A\| \|C\|_p$ *and* $\|CA\|_p \leqslant \|A\| \|C\|_p$.

(iii) *If* $k \geqslant p$, $k \in \mathbb{N}^*$ *and if* $-1 \notin \sigma(C)$, *the infinite product*

$$\delta_k(C) := \prod_{i=1}^{\infty} \left((1+\lambda_i) \exp\left(\sum_{j=1}^{k-1} (-1)^j \frac{\lambda_i^j}{j}\right)\right)$$

converges absolutely.

(iv) *If* $k \geqslant p \geqslant k-1$, $(-1 \notin \sigma(C))$ *there exists a constant* K_1 *depending only on* p *such that* $|\delta_k(C)| \leqslant \exp(K_1 \|C\|_p^p)$.

(v) *For each* $C_1 \in \mathcal{C}_p$, *the function* $\delta_k(C + zC_1)$ *is an analytic function of* z.

THEOREM 2.5. *Let* $1 \leqslant p < \infty$ *and* $C \in \mathcal{C}_p$ *such that* $-1 \notin \sigma(C)$.

Let $k \geqslant p \geqslant k-1$, *and let* $\delta_k(C)$ *be the infinite product defined previously. Then the operator* $\delta_k(C)(\mathrm{Id}+C)^{-1}$ *depends continuously on* C, *and satisfies the inequality:*

$$\|\delta_k(C)(\mathrm{Id}+C)^{-1}\| \leqslant \exp(K_2 \|C\|_p^p)$$

where K_2 *is a constant depending only on* p.

3. RESULTS

First of all, we give a Lemma (see Lemma 1 in [7]) which is very useful in the sequel:

LEMMA 3.1. *Let* $f : B(0,1) \to \Omega$ *be a biholomorphic map of the unit disk onto an open simply connected bounded set* Ω *with a* C^2 *regular boundary. Then there exist two positive constants* c *and* C *such that:*

$$c(1-|z|) \leqslant d(f(z), \partial\Omega) \leqslant C(1-|z|).$$

Now we give the first theorem, which can be considered as a generalization of Theorem 3.5 of [1]. In [1] one considers certain compact perturbations of an operator A whose spectrum is contained in a finite union of simple rectifiable smooth curves J ($\sigma(A)$ can be a singleton) and whose resolvent has the following polar growth condition near J:

$$\exists\, K, n > 0; \quad \|(\lambda \mathrm{Id} - A)^{-1}\| \leqslant \frac{K}{d(\lambda, J)^n}.$$

THEOREM 3.2. *Let $A \in \mathcal{L}(\mathcal{H}, A_e)$ and let J (not reduced to a single point) be an exposed arc of $\sigma(A)$ such that the resolvent of A satisfies the following growth condition near J:*

$$\|(\lambda \mathrm{Id} - A)^{-1}\| \leqslant \varphi(d(\lambda, J))$$

where φ is a positive decreasing function satisfying $\int_0^\varepsilon \log \varphi(x)\, dx < \infty$ for some sufficiently small $\varepsilon > 0$. Then, if $B \in \mathcal{C}_p$, $1 \leqslant p < \infty$, the operator $T := A + B$ has proper hyperinvariant subspaces.

Proof. Without loss of generality for the existence of hyperinvariant subspaces, we may assume that J is an exposed arc of $\sigma(T)$. In fact, we can suppose $\sigma_p(T) = \emptyset$. By Weyl's theorem (Theorem 0.10 in [8]) $\sigma(T) \subset \sigma(A)$ and it is clear that $\sigma_e(T) = \sigma_e(A)$. Since J consists of accumulation points of $\partial\sigma(A)$ (see for example Corollary 1.26 of [6]), J is a part of $\sigma_e(A)$. So we get $J \subset \sigma_e(A) = \sigma_e(T) \subset \sigma(T) \subset \sigma(A)$ and we easily deduce from this relation that J is an exposed arc of $\sigma(T)$.

Now we will prove that $R(\cdot\,; T)$ satisfies the growth condition (C) near J. Since J is an exposed arc for $\sigma(T)$ and $\sigma(A)$, if $\lambda \in \vartheta(J) \cap \rho(A)$, we get $d(\lambda, J) = d(\lambda, \sigma(A)) = d(\lambda, \sigma(T))$ (where $\vartheta(J)$ denotes a neighborhood of J).

Let $\lambda \in \rho(A)$, λ near J and set $d(\lambda) = d(\lambda, J)$. It is clear that $(\lambda \mathrm{Id} - T)^{-1}$ is well-defined and that $(\lambda \mathrm{Id} - T)^{-1} = (\lambda \mathrm{Id} - A)^{-1}(\mathrm{Id} - B(\lambda \mathrm{Id} - A)^{-1})^{-1}$. In particular $-1 \notin \sigma(-B(\lambda \mathrm{Id} - A)^{-1})$. Upon setting $k = \mathrm{Ent}(p) + 1$, by Theorem 2.4 (iii), we obtain that the function $\delta_k(-B(\lambda \mathrm{Id} - A)^{-1})$ does not vanish. So we get:

$$(\lambda \mathrm{Id} - T)^{-1} = \frac{(\lambda \mathrm{Id} - A)^{-1}\delta_k(-B(\lambda \mathrm{Id} - A)^{-1})(\mathrm{Id} - B(\lambda \mathrm{Id} - A)^{-1})^{-1}}{\delta_k(-B(\lambda \mathrm{Id} - A)^{-1})}.$$

By Theorem 2.3 and Theorem 2.4 (ii), we get:

$$\begin{aligned}\|\delta_k(-B(\lambda \mathrm{Id} - A)^{-1})(\mathrm{Id} - B(\lambda \mathrm{Id} - A)^{-1})^{-1}\| &\leqslant \exp(K_2 \|B(\lambda \mathrm{Id} - A)^{-1}\|_p^p) \\ &\leqslant \exp(K_2 \|B\|_p^p \varphi(d(\lambda))^p).\end{aligned}$$

In order to estimate $\delta_k(-B(\lambda \mathrm{Id} - A)^{-1})$ we use the Borel-Carathéodory theorem. This result enables us to deduce an upper bound for the modulus of a function on a circle $|z| = r$, from a bound for its real part on a larger concentric circle $|z| = R$.

THEOREM 3.3. (Borel-Carathéodory) *Let $f(z)$ be an analytic function regular for $|z| \leqslant R$, and let $M(r)$ and $A(r)$ denote the maxima of $|f(z)|$ and $\Re(f(z))$ on $|z| = r$. Then for $0 < r < R$:*

$$M(r) \leqslant \frac{2r}{R-r} A(R) + \frac{R+r}{R-r} |f(0)|.$$

Before using this last theorem we have to do some preliminary work. Let $\Delta = [\alpha, \beta]$ be any segment of the curve J, where α, β are not singular points of J (α precedes β in a positive direction along J). Let us replace J by $[\alpha, \beta]$. We define two simply connected domains D_1 and D_2, whose boundaries are simply rectifiable and such that $\partial D_1 \cap \partial D_2 = [\alpha, \beta]$ (cf. Figure 1). We denote ∂D_i by C_i ($i = 1, 2$).

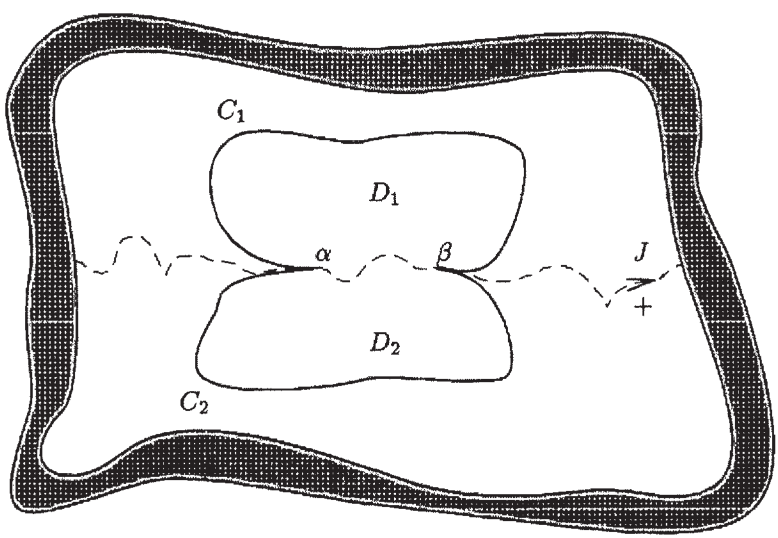

Figure 1.

Set $\delta(\lambda) = \delta_k(-B(\lambda \mathrm{Id} - A)^{-1})$. This function is well defined and analytic for all λ in $\rho(A)$. Moreover,

$$|\delta(\lambda)| \leqslant \exp(K_1 \|B\|_p^p \varphi(d(\lambda))^p), \ \delta(\lambda) \neq 0, \ \lambda \in \rho(A).$$

Indeed, by Theorem 2.4 (ii) and (iv) we have

$$\begin{aligned}|\delta_k(-B(\lambda\mathrm{Id}-A)^{-1})| &\leqslant \exp(K_1\|B(\lambda\mathrm{Id}-A)^{-1}\|_p^p)\\ &\leqslant \exp(K_1\|B\|_p^p\|(\lambda\mathrm{Id}-A)^{-1}\|^p)\\ &\leqslant \exp(K_1\|B\|_p^p\varphi(d(\lambda))^p).\end{aligned}$$

Therefore we get $\delta(\lambda)=\exp(\alpha_j(\lambda))$ where $\alpha_j(\lambda)$ is a bounded analytic function on D_j, $j=1,2$. Thus,

$$\Re\alpha_j(\lambda)\leqslant K_1\|B\|_p^p\varphi(d(\lambda))^p.$$

Let $z=\Phi_j(\lambda)$ be the conformal mapping of the domain D_j onto the unit disc and let $\lambda=\Psi_j(z)$ be the reciprocal mapping for $j=1,2$ respectively. Then $\alpha_j(\Psi_j(z))$ are functions analytic in the unit disc, which satisfy the inequality:

$$\begin{aligned}\Re\alpha_j(\lambda) &= \Re\alpha_j(\Psi_j(z))\\ &\leqslant K_1\|B\|_p^p\varphi(d(\lambda))^p\\ &\leqslant K_1\|B\|_p^p\varphi(C_j(1-|z|))^p\end{aligned}$$

for some constant $C_j>0$ ($j=1,2$), thanks to Lemma 3.1 and the fact that φ is decreasing. Let r be an arbitrary real value satisfying $1/2\leqslant r<1$, and let $|z|<r$. Using the Borel-Carathéodory inequality, we obtain:

$$|\alpha_j(\Psi_j(z))|\leqslant\frac{2|z|}{r-|z|}K_1\|B\|_p^p\varphi(C_j(1-r))^p+\frac{r+|z|}{r-|z|}|\alpha_j(\Psi_j(0))|.$$

If z satisfies $|z|=2r-1$, we get:

$$|\alpha_j(\Psi_j(z))|\leqslant\frac{D_1}{1-|z|}\varphi\left(\frac{C_j}{2}(1-|z|)\right)^p+\frac{D_2}{1-|z|}$$

for some constant $D_j>0$ ($j=1,2$).

We recall that for any operator $L\in\mathcal{L}(\mathcal{H})$ we have:

$$\|(\lambda\mathrm{Id}-L)^{-1}\|\geqslant\frac{1}{d(\lambda,\sigma(L))}.$$

So, obviously, under the assumption that $|z|$ is near 1, we get the following inequality:

$$\frac{1}{1-|z|}\leqslant k\varphi\left(\frac{C_j}{2}(1-|z|)\right)$$

where k is a positive constant. Hence, we obtain:

$$|\alpha_j(\Psi_j(z))|\leqslant K\varphi\left(\frac{C_j}{2}(1-|z|)\right)^{p+1}$$

for some constant $K > 0$. Thus, returning back to the domain D_1 and D_2 respectively and using once more Lemma 3.1 and the monotony of the function φ, we find that:

$$|\alpha_j(\lambda)| \leqslant D\varphi(D'd(\lambda))^{p+1}$$

for some positive constants D and D'. In particular,

$$\Re\alpha_j(\lambda) \geqslant -D\varphi(D'd(\lambda))^{p+1},$$

so that:

$$|\delta(\lambda)|^{-1} \leqslant \exp(D\varphi(D'd(\lambda))^{p+1}).$$

It follows from that:

$$\|(\lambda\mathrm{Id} - T)^{-1}\| \leqslant \varphi(d(\lambda))\exp(K_1\|B\|_p^p\varphi(d(\lambda))^p)\exp(D\varphi(D'd(\lambda))^{p+1}).$$

Therefore we have:

$$\|(\lambda\mathrm{Id} - T)^{-1}\| \leqslant \exp(M_1\varphi(M_2 d(\lambda))^{p+1})$$

for some positive constants M_1 and M_2 depending only on p. We easily verify that:

$$\int_0 \log\log\exp(M_1\varphi(M_2\rho)^{p+1})\,\mathrm{d}\rho < \infty.$$

The proof of the theorem is now completed thanks to the Ljubič-Macaev's theorem. ∎

The following corollary presents an explicit majorant of the growth of the resolvent which is stronger that the one proposed in Theorem 3.5 of [1].

COROLLARY 3.4. *Let $A \in \mathcal{L}(\mathcal{H}, A_e)$ and let J (not reduced to a single point) be an exposed arc of $\sigma(A)$ such that the resolvent of A satisfies the following growth condition near J:*

$$\exists\, K_1', K_2' > 0,\ \varepsilon > 0;\quad \|(\lambda\mathrm{Id} - A)^{-1}\| \leqslant K_1' \exp\left(\frac{K_2'}{d(\lambda, J)|\log d(\lambda, J)|^{1+\varepsilon}}\right).$$

Then, if $B \in \mathcal{C}_p$, $1 \leqslant p < \infty$, the operator $T := A + B$ has proper hyperinvariant subspaces.

The following theorem is a generalization of Theorem 3.8 in [1] where, as mentioned before, the resolvents have polynomial growth conditions near J_k.

THEOREM 3.5. *Let $T_k \in \mathcal{L}(\mathcal{H}, A_e)$ such that $\sigma(T_k) \subset J_k$ where J_k, $(k = 1,2)$ is a finite union of simple rectifiable smooth Jordan arcs, and suppose $R(\cdot\,; T_k)$ has the following growth condition near J_k:*

$$\|(\lambda \mathrm{Id} - T_k)^{-1}\| \leqslant \varphi_k(d(\lambda, J_k))$$

where φ_k is a positive decreasing function defined on positive reals and satisfying $\int_0^{\varepsilon} \log \varphi_k(x)\,\mathrm{d}x < \infty$, for ε sufficiently small, $k = 1,2$.

Then, if $T_1T_2 \in \mathcal{C}_p$, $1 \leqslant p < \infty$ and $\alpha_k \in \mathbf{C}$ $(k = 1,2)$, the operator $T := \alpha_1 T_1 + \alpha_2 T_2$ has non-trivial hyperinvariant subspaces whenever $\sigma(\alpha_1 T_1 + \alpha_2 T_2)$ is not reduced to a singleton.

Before detailing the proof of this theorem, we give the following lemma.

LEMMA 3.6. *Let $T, S \in \mathcal{L}(\mathcal{H})$ such that $TS \in \mathcal{K}(\mathcal{H})$. Then we have*

$$\sigma_c(T + S) \subset \sigma(T) \cup \sigma(S).$$

Proof. Suppose this assertion does not hold. Then we can find $\lambda \in \sigma_c(T+S)$ such that $\lambda \in \rho(T) \cap \rho(S)$. Thus, we can find $S(\lambda), T(\lambda) \in \mathcal{L}(\mathcal{H})$ such that:

$$S(\lambda)T(\lambda)(\lambda - T)(\lambda - S) = \mathrm{Id}.$$

Since TS is compact, TS can not be invertible, which implies $\lambda \neq 0$. We have $\lambda(\lambda - T - S) = (\mathrm{Id} - TSS(\lambda)T(\lambda))(\lambda - T)(\lambda - S)$. Since $\lambda \in \sigma_c(T+S)$, there exists a sequel in $\mathcal{H}$ $(x_n)_{n \geqslant 1}$ such that $\|x_n\| = 1$ and $\lim\limits_{n\to\infty} (\lambda - T - S)x_n = 0$. We get $\lim\limits_{n\to\infty} (\mathrm{Id} - TSS(\lambda)T(\lambda))y_n = 0$ with $y_n = (\lambda - T)(\lambda - S)x_n$. Since TS is compact, by dropping to a subsequence if necessary, we may assume that $(TSS(\lambda)T(\lambda)y_n)_{n\geqslant 1}$ is convergent. Consequently, $(y_n)_{n \geqslant 1}$ is convergent. If $y = \lim\limits_{n\to\infty} y_n$ one obtains $S(\lambda)T(\lambda)y = \lim\limits_{n\to\infty} x_n$ and hence $(\lambda - T - S)S(\lambda)T(\lambda)y = 0$, $\|S(\lambda)T(\lambda)y\| = 1$, contradicting the assumption $\lambda \in \sigma_c(T + S)$. ∎

Now we can give the proof of the theorem.

Proof of Theorem 3.5. Let us set $\alpha_1 J_1 \cup \alpha_2 J_2 = J$, $\varepsilon = \min(\varepsilon_1, \varepsilon_2)$ and $d(\lambda) := d(\lambda, J)$. Without loss of generality for the existence of hyperinvariant subspaces, we can suppose $\sigma(\alpha_1 T_1 + \alpha_2 T_2) = \sigma_c(\alpha_1 T_1 + \alpha_2 T_2)$ is connected. By Lemma 3.6, we have $\sigma_c(\alpha_1 T_1 + \alpha_2 T_2) \subset \sigma(\alpha_1 T_1) \cup \sigma(\alpha_2 T_2) \subset J$, thus we get $\alpha_1 T_1 + \alpha_2 T_2$ belongs to $\mathcal{L}(\mathcal{H}, A_e)$. Let us set $V(\lambda) = -\alpha_1\alpha_2 R(\lambda; \alpha_2 T_2) R(\lambda; \alpha_1 T_1) T_1 T_2$, $\lambda \notin J$. We have $\lambda(\lambda - \alpha_1 T_1 - \alpha_2 T_2) = (\lambda - \alpha_1 T_1)(\lambda - \alpha_2 T_2)(\mathrm{Id} + V(\lambda))$, thus $-1 \notin \sigma(V(\lambda))$ and by the definition of $V(\cdot)$ we infer also that:

$$\|V(\lambda)\|_p \leqslant C\varphi_1(d(\lambda))\varphi_2(d(\lambda))$$

for some positive constant C. We now proceed along the lines of the proof of Theorem 3.2. We first get upper bounds for $\|\delta_k(V(\lambda))(\mathrm{Id}+V(\lambda))^{-1}\|$ and $\|\delta_k(V(\lambda))^{-1}\|$. Then, via the equality

$$R(\lambda, \alpha_1 T_1 + \alpha_2 T_2) = \lambda \delta_k(V(\lambda))(\mathrm{Id} + V(\lambda))^{-1} R(\lambda, \alpha_1 T_1) R(\lambda, \alpha_2 T_2) \delta_k(V(\lambda))^{-1}$$

(where $k = \mathrm{Ent}(p) + 1$) we obtain:

$$\|R(\lambda, \alpha_1 T_1 + \alpha_2 T_2)\| \leqslant \exp\left(C_1(\varphi_1 \varphi_2(C_2 d(\lambda)))^{1+p}\right)$$

for some positive constant C_1 and C_2. Now we can apply Theorem 2.3 to finish the proof. ∎

REMARK 3.7. One may think of improving the above Theorem 3.2 by "relaxing" the smoothness condition on the exposed arc. In fact, Professor Macaev (private communication) has pointed out to us that the Ljubič-Macaev theorem (Theorem 2.3) would hold under somewhat weaker than C^2-smoothness (loosely speaking something like "finite curvature"). Nevertheless, via our method, there still remains the question of how much can be "decreased" the smoothness hypothesis in Lemma 3.1.

Acknowledgements. The author would like to thank the referee for pertinent questions and remarks leading to improvements of the paper. The author expresses also her gratitude to E. Amar, R. Gay and P. Thomas for stimulating discussions on the regularity of the exposed arc.

REFERENCES

1. C. APOSTOL, On the growth of resolvent, perturbation and invariant subspaces. *Rev. Roumaine Math. Pures Appl.* **16**(1971), 345–350.
2. I. COLOJOARĂ, C. FOIAŞ, *Theory of Generalized Spectral Operators*, Gordon & Breach, New York 1968.
3. N. DUNFORD, V.T. SCHWARTZ, *Linear Operators*, vol. II, Interscience, New York 1963.
4. K. KITANO, Invariant subspaces of some non-selfadjoint operators, *Tôhoku Math. J. (2)* **20**(1968), 313–322.
5. JU.I LJUBIČ, V.I MACAEV, On operators with a separable spectrum, *Amer. Math. Soc. Transl. Ser. (2)* **47**(1965), 89–129.
6. C. PEARCY, *Some recent developments in operator theory*, CBMS Regional Conf. Ser. in Math., vol. 36, Amer. Math. Soc., Providence, 1978.
7. M. RADJABALIPOUR, On decomposability of compact perturbations of operators, *Proc. Amer. Math. Soc.* **53**(1975)159–164.

8. H. RADJAVI, P. ROSENTHAL, *Invariant Subspaces*, Springer-Verlag, Berlin – Heidelberg – New York 1973.

ISABELLE CHALENDAR
U.F.R Mathématiques et Informatique
Université Bordeaux I
351, Cours de la Libération
33405 Talence Cedex
FRANCE

Received July 29, 1995; revised June 7, 1996.