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SUBALGEBRAS OF REFLEXIVE ALGEBRAS—ERRATUM

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The proof of Proposition 4.1(4) of [1] does not establish the asserted result. What the proof demonstrates is the following:

PROPOSITION 1. *If* $\{\mathscr{A}_i : i \in I\}$ *is a collection of* σ*-weakly closed unital algebras and the direct sum of the* $\mathscr{A}_i$ *has property* D_σ*, then there is an* $r \geqslant 1$ *such that* $\mathscr{A}_i$ *has property* $\mathrm{D}_\sigma(r)$ *for all but finitely many i in I.*

COROLLARY 2. *If* $\mathscr{A}_i$ *has property* $\mathrm{D}_\sigma(r_i)$ *for every i in I and the direct sum of the* $\mathscr{A}_i$ *has property* D_σ*, then there is an* $r \geqslant 1$ *such that* $\mathscr{A}_i$ *has property* $\mathrm{D}_\sigma(r)$ *for every i in I.*

COROLLARY 3. *A* σ*-weakly closed unital algebra* $\mathscr{A}$ *has property* $\mathrm{D}_\sigma(r)$ *for some* $r \geqslant 1$ *if and only if* $\mathscr{A} \oplus \mathscr{A} \oplus \ldots$ *has property* D_σ.

Thus, the application of Proposition 4.1(4) in Theorem 5.9 of [1] is valid. However, the last sentence of part (5) in Section 6 (Questions and Comments) should be deleted.

We wish to point out that the direct integral version of Proposition 4.1(4) is true as long as the σ-finite measure involved is nonatomic.

THEOREM 4. *Suppose* $(X, \mathscr{M}, \mu)$ *is a nonatomic* σ*-finite measure space,* $\{A_x : x \in X\}$ *is a measurable family of* σ*-weakly closed unital algebras, and* $\mathscr{A} = \int_X^{\oplus} \mathscr{A}_x \,\mathrm{d}\mu(x)$. *If* $\mathscr{A}$ *has property* D_σ*, then there is an* $r \geqslant 1$ *such that* $\mathscr{A}$ *has property* $\mathrm{D}_\sigma(r)$.

Proof. Without loss of generality, we may assume $\mu(X) = 1$. If E is a measurable subset of X, then let $\mathscr{A}_E = \int_E^{\oplus} \mathscr{A}_x \,\mathrm{d}\mu(x)$. Then $\mathscr{A} = \mathscr{A}_E \oplus \mathscr{A}_{X \setminus E}$. Call an algebra $\mathscr{B}$ "bad" if there is no $r \geqslant 1$ such that $\mathscr{B}$ has property $\mathrm{D}_\sigma(r)$. Suppose $\mathscr{A}$ has property D_σ, but $\mathscr{A}$ is bad. We will show that this leads to a contradiction. Since μ is nonatomic and $\mu(X) = 1$, there exist disjoint measurable sets E and F such that

$\mu(E) = \mu(F) = 1/2$. Since $\mathscr{A} = \mathscr{A}_E \oplus \mathscr{A}_F$, it follows that either $\mathscr{A}_E$ or $\mathscr{A}_F$ is bad. Hence there is a measurable subset E_1 such that $\mathscr{A}_{E_1}$ is bad and $\mu(E_1) = 1/2$. Proceeding inductively, we obtain a sequence $E_1 \supset E_2 \supset \ldots$ of measureable sets such that $\mathscr{A}_{E_n}$ is bad and $\mu(E_n) = 1/2^n$. Let $F_n = E_n \setminus E_{n+1}$ for $n = 1, 2, \ldots$. Then $\sum_{n=1}^{\infty} \oplus \mathscr{A}_{F_n}$ has property D_σ, and it follows from Proposition 1 that there is an integer $N \geqslant 1$ and an $r \geqslant 1$ such that $\mathscr{A}_{F_n}$ has property $\mathrm{D}_\sigma(r)$ for all $n \geqslant N$. Thus $\sum_{n=N}^{\infty} \oplus \mathscr{A}_{F_n}$ has property $\mathrm{D}_\sigma(r)$. However, since $\mu\left(\bigcap_{n=1}^{\infty} E_n\right) = 0$, it follows that $\sum_{n=N}^{\infty} \oplus \mathscr{A}_{F_n} = \mathscr{A}_{E_n}$, which is a contradiction since $\mathscr{A}_{E_n}$ was chosen to be bad.

COROLLARY 5. *If $\mathscr{A}$ is an ultraweakly closed algebra whose center contains a commutative nonatomic von Neumann algebra, and if $\mathscr{A}$ has property D_σ, then $\mathscr{A}$ has property $\mathrm{D}_\sigma(r)$ for some r.*

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REFERENCE

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