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ON THE LIFTING PROPERTY FOR UNIVERSAL C^*-ALGEBRAS OF OPERATOR SPACES

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Abstract. Operator spaces whose universal C^*-algebras have the local lifting property are studied.

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1. INTRODUCTION AND PRELIMINARY BACKGROUND

The universal C^*-algebra of an operator space X, which will be denoted by $C^*\langle X\rangle$, is defined in the next statement.

Theorem 1.1. (Theorem 3.2 in [15]) *Let X be an operator space. There exist a C^*-algebra $C^*\langle X\rangle$ and a completely isometric embedding $i : X \to C^*\langle X\rangle$ which have the following properties:*

(i) *The image $i(X)$ generates $C^*\langle X\rangle$ as a C^*-algebra.*

(ii) *For any C^*-algebra B and any complete contraction $\varphi : X \to B$, there exists a $*$-homomorphism $\theta : C^*\langle X\rangle \to B$ such that $\theta \circ i = \varphi$.*

Moreover, such a pair $(C^*\langle X\rangle, i)$ is essentially unique, i.e. if (A, j) is another pair with the above property, then there exists a $*$-isomorphism $\rho : A \to C^*\langle X\rangle$ with $\rho \circ j = i$.

We will identify X as a subspace of $C^*\langle X\rangle$ and omit i and denote the unitization of $C^*\langle X\rangle$ by $C^*\langle X\rangle^+$. We denote the set of all bounded operators on a separable infinite dimensional Hilbert space by $\mathbb{B}$. Throughout this paper, B means a C^*-algebra and J means a closed two-sided ideal in B. We denote this by $J \triangleleft B$ and denote the quotient map by $\pi : B \to B/J$.

The lifting property and the local lifting property have been defined by Kirchberg ([11]).

DEFINITION 1.2. ([11]) Let A be a unital C^*-algebra. We say A has the *Local Lifting Property* (LLP) if for any $J \triangleleft B$, any unital complete positive map $\varphi : A \to B/J$ and any finite dimensional operator system $E \subset A$, the restriction of φ to E has a unital completely positive lifiting $\psi : E \to B$ (i.e., $\pi \circ \psi = \varphi|E$). We say A has the *Lifting Property* (LP) if one can take $E = A$ in the above situation. In case A is a non-unital C^*-algebra, we say A has the LLP (respectively the LP) if the unitization A^+ of A has the LLP (respectively the LP).

It is known that the full group C^*-algebra $C^*(\mathbb{F})$ of any countable free group $\mathbb{F}$ has the LP. If $\mathbb{F}$ is uncountable, then $C^*(\mathbb{F})$ has the LLP ([11]). In [11], E. Kirchberg found various pairs of C^*-algebras such that there is only one C^*-norm on the algebraic tensor product of each of them. In particular,

THEOREM 1.3. (Proposition 1.1 in [11], see also [17]) *Let A be a C^*-algebra, then $A \otimes_{\max} \mathbb{B} = A \otimes_{\min} \mathbb{B}$ if and only if A has the* LLP.

Now, we see how universal C^*-algebras are related to the LLP. Let X be an operator space. We denote the algebraic tensor product of X and $\mathbb{B}$ by $X \otimes \mathbb{B}$. Then for $y \in X \otimes \mathbb{B}$, the norm of y induced by $C^*\langle X\rangle \otimes_{\max} \mathbb{B}$ is $\sup\{\|(\theta \cdot \rho)(y)\|\}$, where the supremum is taken over all Hilbert spaces H and pairs of $*$-representations θ of $C^*\langle X\rangle$ and ρ of $\mathbb{B}$ whose ranges commute. Since each $*$-representation θ of $C^*\langle X\rangle$ is determined by the complete contraction $\theta|X$, this norm is equal to the δ-norm defined below.

DEFINITION 1.4. Let X be an operator space. For $y \in X \otimes \mathbb{B}$, we define the δ-norm by

$$\delta(y) = \sup\{\|(\sigma \cdot \rho)(y)\|_{\mathbb{B}(H)}\},$$

where the supremum is taken over all Hilbert spaces H and all pairs (σ, ρ) where $\rho : \mathbb{B} \to \mathbb{B}(H)$ is a $*$-representation and $\sigma : X \to \mathbb{B}(H)$ is a complete contraction whose ranges commute. We denote the resulting (after completion) Banach space by $X \otimes_\delta \mathbb{B}$. Then

$$X \otimes_\delta \mathbb{B} \subset C^*\langle X\rangle \otimes_{\max} \mathbb{B} \text{ isometrically.}$$

On the other hand, the norm on $X \otimes \mathbb{B}$ induced by $C^*\langle X\rangle \otimes_{\min} \mathbb{B}$ is, of course, the minimal norm. So, if $C^*\langle X\rangle$ has the LLP, from Theorem 1.3 we have $X \otimes_\delta \mathbb{B} = X \otimes_{\min} \mathbb{B}$. We will show that the converse is also true. This is an operator space analogue of Theorem 1.3.

Pisier's Theorem given below states that the δ-norm is a suitable factorization norm.

THEOREM 1.5. (Theorem 6.3.1 and Corollary 6.3.5 in [18]) *Let $y \in X \otimes \mathbb{B}$ and $\widetilde{y} : X^* \to \mathbb{B}$ be the associated map. Then we have*

$$\delta(y) = \inf\{\|u\|_{\mathrm{cb}}\,\|v\|_{\mathrm{cb}}\}$$

where the infimum is taken over all n, maps $v : X^* \to \mathbb{M}_n$, and $u : \mathbb{M}_n \to \mathbb{B}$ with v weak-continuous and $\widetilde{y} = u \circ v$.*

At the end of this section, we state the revived Kaplansky density theorem.

LEMMA 1.6. (Lemma 2.1 in [4]) *Let R_* be the predual of a von Neumann algebra R and let B be a C^*-algebra. Then every complete contraction from R_* to B^{**} (respectively B) can be approximated by finite rank complete contractions from R_* to B in the point-weak* (respectively point-norm) topology.*

2. MAIN RESULTS

DEFINITION 2.1. Let X be an operator space. For $\lambda > 0$, we say X has the λ-OLLP if given any $J \triangleleft B$ and a complete contraction $\varphi : X \to B/J$, for every finite dimensional subspace E of X, there exists a map $\psi : E \to B$ with cb-norm $\leqslant \lambda$ such that $\pi \circ \psi = \varphi|E$. We say X has the λ-OLP if one can take $E = X$ in the above situation. We say X has the OLLP (respectively OLP) if X has the λ-OLLP (respectively λ-OLP) for some $\lambda > 0$.

PROPOSITION 2.2. *The operator space X has the* 1-OLLP *(respectively the* 1-OLP *and is separable) if and only if $C^*\langle X\rangle$ has the* LLP *(respectively the* LP *and is separable).*

Proof. We only prove the case when X has the 1-OLLP. Assume that X has the 1-OLLP. To check that $C^*\langle X\rangle$ has the LLP, we give ourselves $J \triangleleft B$, a unital completely positive map $\varphi : C^*\langle X\rangle^+ \to B/J$ and a finite dimensional operator system $E \subset C^*\langle X\rangle^+$. By a standard approximation argument, we may assume that there is a finite dimensional subspace $F \subset X$ such that E is contained in the C^*-subalgebra of $C^*\langle X\rangle^+$ generated by F and the unit. Note that this C^*-subalgebra is canonically $*$-isomorphic to $C^*\langle F\rangle^+$. We fix a surjective unital $*$-homomorphism $\rho : C^*(\mathbb{F}) \to C^*\langle X\rangle^+$. By the assumption, there exists a complete contraction $\sigma : F \to C^*(\mathbb{F})$ such that $\rho \circ \sigma = \mathrm{id}_F$. By universality, σ extends to a unital $*$-homomorphism $\theta : C^*\langle F\rangle^+ \to C^*(\mathbb{F})$. Clearly, we have $\rho \circ \theta = \mathrm{id}_{C^*\langle F\rangle^+}$. Applying the LLP of $C^*(\mathbb{F})$ to $\varphi \circ \rho$, we obtain a unital completely positive map $\psi : \theta(E) \to B$ such that $\pi \circ \psi = \varphi \circ \rho|\theta(E)$. Then, $\psi \circ \theta|E$ is a unital completely positive lifting of φ. This proves that $C^*\langle X\rangle$ has the LLP. The "if" part is easy. ∎

The lemma below is due to Arveson ([1]). In fact, it is proved there for operator systems and unital completely positive maps, but it can be generalized to the case of operator spaces and completely bounded maps in a formal way.

LEMMA 2.3. *Let X be a separable operator space and let $J \triangleleft B$. Then the set of all completely contractively liftable maps from X to B/J is closed in the point-norm topology.*

Proof. Combine Theorem 6 in [1] and Lemma 7.1 in [13]. ∎

COROLLARY 2.4. *Let X be a separable operator space. If X has the λ_1-*OLLP *and the λ_2-completely bounded approximation property, then X has the $\lambda_1\lambda_2$-*OLP.

Proof. We give ourselves a complete contraction $\varphi : X \to B/J$. Let $E \subset X$ be any finite dimensional space and let $\varepsilon > 0$ be arbitrary. By the assumption, there is a finite rank map ρ on X with $\|\rho\|_{\mathrm{cb}} < \lambda_2 + \varepsilon$ such that $\rho|E = \mathrm{id}_E$. Since X has the λ_1-OLLP, there is a map $\psi' : \rho(X) \to B$ with $\|\psi'\|_{\mathrm{cb}} \leqslant \lambda_1$ such that $\pi \circ \psi' = \varphi|\rho(X)$. Define $\psi : X \to B$ by $\psi = \psi' \circ \rho$. Then, we have $\|\psi\|_{\mathrm{cb}} < \lambda_1(\lambda_2 + \varepsilon)$ and $\pi \circ \psi|E = \varphi|E$. By Lemma 2.3, we are done. ∎

Let $E \subset \mathbb{M}_n$ and let $J \triangleleft B$. Since for any $x \in E \otimes_{\min} B$, we have

$$\|x + \mathbb{M}_n \otimes_{\min} J\| = \lim \|x(1 \otimes (1 - e_n))\| = \|x + E \otimes_{\min} J\|,$$

where $\{e_i\}_i$ is any approximate unit for J, we have a canonical isometric inclusion

$$(E \otimes_{\min} B)/(E \otimes_{\min} J) \subset (\mathbb{M}_n \otimes_{\min} B)/(\mathbb{M}_n \otimes_{\min} J).$$

Since

$$\mathbb{M}_n \otimes_{\min} (B/J) = (\mathbb{M}_n \otimes_{\min} B)/(\mathbb{M}_n \otimes_{\min} J),$$

we have

$$E \otimes_{\min} (B/J) = (E \otimes_{\min} B)/(E \otimes_{\min} J).$$

By [3], [5], for any operator space X, we have the canonical isometric identification $E \otimes_{\min} X = \mathrm{CB}(E^*, X)$. Using Lemma 2.3, we conclude that any complete contraction φ from E^* to any quotient C^*-algebra B/J has a completely contractive lifting $\psi : E^* \to B$, i.e., $\pi \circ \psi = \varphi$. Now, let F be a finite dimensional operator space. For any $J \triangleleft B$, there is a canonical contractive isomorphism

$$T : (F \otimes_{\min} B)/(F \otimes_{\min} J) \to F \otimes_{\min} (B/J).$$

We define the exactness constant $\mathrm{ex}(F)$ of F by $\mathrm{ex}(F) = \sup\{\|T^{-1}\|\}$, where the supremum is taken over all C^*-algebras B and ideals $J \triangleleft B$. By the same argument as above, we conclude that for a finite dimensional operator space F, we have $\mathrm{ex}(F) \leqslant \lambda$ if and only if any complete contraction φ from F^* to any quotient C^*-algebra B/J has a lifting with cb-norm $\leqslant \lambda$. See [16] for more information about the exactness constant.

Theorem 2.5. *For an operator space X, the following are equivalent:*

(i) *The operator space X has the* λ-OLLP.

(ii) *The formal identity map $X \otimes_{\min} \mathbb{B} \to X \otimes_{\delta} \mathbb{B}$ has norm $\leqslant \lambda$.*

(iii) *For any finite dimensional subspace $E \subset X$ and any $\varepsilon > 0$, there exist a finite dimensional operator space F and two maps $\beta : E \to F$, $\alpha : F \to X$ such that F^* is a subspace of a full matrix algebra, $\alpha \circ \beta = \mathrm{id}_E$ and $\|\alpha\|_{\mathrm{cb}} \|\beta\|_{\mathrm{cb}} \leqslant \lambda + \varepsilon$.*

(iv) *For any complete metric surjection q from any operator space Y onto X, any finite dimensional subspace $E \subset X$ and any $\varepsilon > 0$, there exists a finite dimensional subspace $\widetilde{E} \subset Y$ such that $q|\widetilde{E}$ is a $(\lambda + \varepsilon)$-complete metric surjection onto E, i.e., for any n and any $e \in \mathbb{M}_n(E)$ with $\|e\| < 1$, there exists $\widetilde{e} \in \mathbb{M}_n(\widetilde{E})$ with $\|\widetilde{e}\| < \lambda + \varepsilon$ such that $\mathrm{id}_{\mathbb{M}_n} \otimes q(\widetilde{e}) = e$.*

Proof. (i) $\Rightarrow$ (ii). Let $y \in X \otimes \mathbb{B}$. Then, there exists a finite dimensional subspace E of X with $y \in E \otimes \mathbb{B}$. Now, consider a surjective $*$-homomorphism $\theta : C^*(\mathbb{F}) \to C^*\langle X \rangle^+$. Since X has the λ-OLLP, there exists a map $\psi : E \to C^*(\mathbb{F})$ with cb-norm $\leqslant \lambda$ such that $\theta \circ \psi$ is the canonical inclusion of E into $C^*\langle X \rangle^+$. Then, $\psi \otimes \mathrm{id}_{\mathbb{B}} : E \otimes_{\min} \mathbb{B} \to C^*(\mathbb{F}) \otimes_{\min} \mathbb{B}$ also has cb-norm $\leqslant \lambda$ and $\theta \otimes \mathrm{id}_{\mathbb{B}} : C^*(\mathbb{F}) \otimes_{\min} \mathbb{B} = C^*(\mathbb{F}) \otimes_{\max} \mathbb{B} \to C^*\langle X \rangle^+ \otimes_{\max} \mathbb{B}$ is contractive. Hence,

$$\delta(y) = \|y\|_{C^*\langle X \rangle^+ \otimes_{\max} \mathbb{B}} = \|(\theta \otimes \mathrm{id}_{\mathbb{B}}) \circ (\psi \otimes \mathrm{id}_{\mathbb{B}})(y)\| \leqslant \lambda \|y\|_{E \otimes_{\min} \mathbb{B}}.$$

(ii) $\Rightarrow$ (iii). Let $E \subset X$ be a finite dimensional space. Consider the element $y \in X \otimes E^*$ which is associated to the inclusion map of E into X. We may assume that $E^* \subset \mathbb{B}$ and $y \in X \otimes \mathbb{B}$. Then, by assumption, $\delta(y) \leqslant \lambda \|y\|_{\min} = \lambda$. Now we see from Theorem 1.5 that the restriction map from X^* onto E^* factors through a subspace of a full matrix algebra. Taking the dual again, we are done.

(iii) $\Rightarrow$ (i). Fix a complete contraction $\varphi : X \to B/J$ and fix a finite dimensional subspace $E \subset X$. For any $\varepsilon > 0$, there exist F and α, β as in condition (iii). We may assume $\|\alpha\|_{\mathrm{cb}} \leqslant 1$ and $\|\beta\|_{\mathrm{cb}} \leqslant \lambda + \varepsilon$. Since F^* is a subspace of a full matrix algebra, by the preceding remarks, $\varphi \circ \alpha : F \to B/J$ has a lifting ψ with $\|\psi\|_{\mathrm{cb}} \leqslant 1$. Then, $\psi \circ \beta : E \to B$ is a lifting of $\varphi|E$ with cb-norm $\leqslant \lambda + \varepsilon$. By Lemma 2.3, we are done.

(iii) $\Rightarrow$ (iv). We give ourselves a complete metric surjection $q : Y \to X$, a finite dimensional subspace $E \subset X$ and $\varepsilon > 0$. By (iii), there exist a finite dimensional operator space F with $F^* \subset \mathbb{M}_n$ and two maps $\beta : E \to F$, $\alpha : F \to X$ with $\|\alpha\|_{\mathrm{cb}} < 1$, $\|\beta\|_{\mathrm{cb}} \leqslant \lambda + \varepsilon$ such that $\alpha \circ \beta = \mathrm{id}_E$. Let $u \in F^* \otimes X \subset \mathbb{M}_n \otimes X$ be the element corresponding to α. Since q is a complete metric surjection and $\|u\|_{\min} = \|\alpha\|_{\mathrm{cb}} < 1$, there is $\widetilde{u} \in \mathbb{M}_n \otimes Y$ with $\|\widetilde{u}\|_{\min} < 1$ such that $\mathrm{id}_{\mathbb{M}_n} \otimes q(\widetilde{u}) = u$. Let $\widetilde{\alpha} : \mathbb{M}_n^* \to Y$ be the complete contraction corresponding to $\widetilde{u}$. Let $\widetilde{E} \subset Y$ be the finite dimensional subspace defined by $\widetilde{E} = q^{-1}(E) \cap \widetilde{\alpha}(\mathbb{M}_n^*)$. We have to check that $q|\widetilde{E}$ is a $(\lambda + \varepsilon)$-complete metric surjection. Take $e \in E$ with $\|e\| < 1$. Since $F^* \subset \mathbb{M}_n$, $\beta(e) \in F$ extends to $f \in \mathbb{M}_n^*$ with $\|f\| = \|\beta(e)\| < \lambda + \varepsilon$. Let $\widetilde{e} = \widetilde{\alpha}(f)$. Then, $\|\widetilde{e}\| < \lambda + \varepsilon$ and $q(\widetilde{e}) = q \circ \widetilde{\alpha}(f) = \alpha(f|F^*) = \alpha \circ \beta(e) = e$. This proves that q is a $(\lambda+\varepsilon)$-metric surjection. We leave it to the reader to "complete" the proof.

(iv) $\Rightarrow$ (iii). Let $X^* \subset \mathbb{B}(H)$ be a weak*-homeomorphic completely isometric embedding. Then, the restriction map $q : S_1(H) \to X$ is a complete metric surjection. To prove (iii), take a finite dimensional subspace $E \subset X$ and $\varepsilon > 0$. By (iv), there exists a finite dimensional subspace $\widetilde{E} \subset S_1(H)$ such that $q|\widetilde{E}$ is a $(\lambda + \varepsilon)$-complete metric surjection onto E. We may assume that $\widetilde{E} \subset S_1^m$ for some m. Let $F = S_1^m/(\ker(q) \cap S_1^m)$. Then, we have $F^* \subset (S_1^m)^* = \mathbb{M}_m$. Let $\beta : E \to F$ be the map induced by the inclusion $\widetilde{E} \hookrightarrow S_1^m$. Then, we have $\|\beta\|_{\mathrm{cb}} \leqslant \|\mathrm{id} : E \to \widetilde{E}/(\ker(q) \cap \widetilde{E})\|_{\mathrm{cb}} \leqslant \lambda + \varepsilon$. Let $\alpha : F \to X$ be the complete contraction induced by the inclusion $S_1^m \hookrightarrow S_1(H)$. Clearly, we have $\alpha \circ \beta = \mathrm{id}_E$ and $\|\alpha\|_{\mathrm{cb}}\|\beta\|_{\mathrm{cb}} \leqslant \lambda + \varepsilon$. ∎

Corollary 2.6. *Let X and Y be operator spaces and let $\varphi : X \to Y^{**}$ be a complete contraction. If X has the λ-*OLLP*, then, for any finite dimensional subspace E, $\varphi|E$ can be approximated by maps from E to Y with cb-norm $\leqslant \lambda$ in the point-weak* topology.*

Corollary 2.7. *An operator space X is locally reflexive if X^{**} has the* OLLP.

Proposition 2.8. *Let X be a separable operator space. Then X has the* λ-OLP *if and only if for any B, every complete contraction from X to B^{**} can be approximated by maps from X to B with cb-norm $\leqslant \lambda$ in the point-weak* topology.*

Proof. First, we prove the "if" part. Let $J \triangleleft B$ and $\varphi : X \to B/J$ be a complete contraction. Since $B^{**} = (B/J)^{**} \oplus J^{**}$, φ can be regarded as a map $\widehat{\varphi} : X \to B^{**}$. By assumption, there exists a net of maps $\varphi_i : X \to B$ with cb-norm $\leqslant \lambda$ which converges to $\widehat{\varphi}$ in the point-weak* topology. Then, $\pi \circ \varphi_i : X \to B/J$ converges to φ in the point-weak topology. By a standard convexity argument, we can find a net of maps $\psi_j : X \to B$ with cb-norm $\leqslant \lambda$ such that $\pi \circ \psi_j$ converges to φ in the point-norm topology. By Lemma 2.3, there exists a lifting $\psi : X \to B$ with $\|\psi\|_{\mathrm{cb}} \leqslant \lambda$.

Next, we prove the "only if" part. The following argument is essentially due to E. Kirchberg (see the proof of sublemma 2.5.1 in [11]). Let I be the directed set of all finite dimensional subspaces of B^* and let B_I be a C^*-algebra defined by

$$B_I = \Big\{(x_i)_{i\in I} \in \prod_{i\in I} B : \text{ strong}^*\text{-}\lim x_i \text{ exists in } B^{**}\Big\}.$$

Then the $*$-homomorphism $B_I \ni (x_i)_{i\in I} \mapsto \text{strong}^*\text{-}\lim x_i \in B^{**}$ is surjective by Kaplansky density. Let $\varphi : X \to B^{**}$ be a complete contraction. Since X has the λ-OLP, there is a lifting $\psi : X \to B_I$ with $\|\psi\|_{\mathrm{cb}} \leqslant \lambda$. Let $\psi_i : X \to B$ be the i-th coordinate of ψ. Then $\|\psi_i\|_{\mathrm{cb}} \leqslant \lambda$ and the net $\{\psi_i\}_{i\in I}$ converges to φ in the point-weak* topology. ∎

PROPOSITION 2.9. *A separable operator space X has the λ-OLLP if and only if any complete contraction φ from X to the Calkin algebra $\mathbb{B}/\mathbb{K}$ has a lifting with cb-norm $\leqslant \lambda$.*

The "only if" part is an easy consequence of injectivity of $\mathbb{B}$ and of Lemma 2.3. For the proof of the "if" part, we need a technical lemma. Let E be a finite dimensional operator space and let J be an ideal in a separable unital C^*-algebra B. If θ is a unital completely positive map from B to $\mathbb{B}$ with $\theta(J) \subset \mathbb{K}$, then θ induces complete contractions $\dot{\theta} : B/J \to \mathbb{B}/\mathbb{K}$ and $\overset{\vee}{\theta}: (E \otimes_{\min} B)/(E \otimes_{\min} J) \to (E \otimes_{\min} \mathbb{B})/(E \otimes_{\min} \mathbb{K})$.

LEMMA 2.10. *For $u \in E \otimes (B/J)$ we have*

$$\|u\|_{E\otimes_{\min}(B/J)} = \sup_\theta \|\mathrm{id}_E \otimes \dot{\theta}(u)\|_{E\otimes_{\min}(\mathbb{B}/\mathbb{K})}$$

and

$$\|u\|_{(E\otimes_{\min}B)/(E\otimes_{\min}J)} = \sup_\theta \|\overset{\vee}{\theta}(u)\|_{(E\otimes_{\min}\mathbb{B})/(E\otimes_{\min}\mathbb{K})},$$

where each supremum is taken over all unital completely positive maps $\theta : B \to \mathbb{B}$ with $\theta(J) \subset \mathbb{K}$.

Proof. We prove only the second equation. The first equation follows from the second equation. We may assume that E is embedded into a unital C^*-algebra. Suppose that $v \in E \otimes_{\min} B$ and $\|v + E \otimes_{\min} J\| > 1$. Since J is separable, there is a strictly positive element $h \in J$, i.e., $0 \leqslant h \leqslant 1$ and $J = \overline{hJh}$. Let $p_n = \chi_{[\frac{1}{n},1]}(h) \in J^{**}$. Then, p_n is a projection and since $\lim \|(1-p_n)h\| = 0$, we have $\lim \|(1-p_n)x\| = 0$ for all $x \in J$. Let $f_n \in C_0(0,1]$ be the function defined by $f_n(t) = nt$ for $0 \leqslant t \leqslant \frac{1}{n}$ and $f_n(t) = 1$ for $\frac{1}{n} \leqslant t$, and let $h_n = f_n(h)$. Since $h_n p_n = p_n$, we have

$$\begin{aligned}\|(1\otimes(1-p_n))v(1\otimes(1-p_n))\|_{E\otimes_{\min}B^{**}} &\geqslant \|(1\otimes(1-h_n))v(1\otimes(1-h_n))\|_{E\otimes_{\min}B}\\ &\geqslant \|v + E\otimes_{\min} J\| > 1\end{aligned}$$

for all n. For each n, take a unital completely positive map $\theta_n : (1-p_n)B(1-p_n) \to \mathbb{M}_{k_n}$ with
$$\|\mathrm{id}_E \otimes \theta_n((1 \otimes (1-p_n))v(1 \otimes (1-p_n)))\| > 1.$$
Define a unital complete positive map $\theta'_n : B \to \mathbb{M}_{k_n}$ by
$$\theta'_n(b) = \theta_n((1-p_n)b(1-p_n)).$$
Finally, let $\theta : B \to \prod_{n=1}^{\infty} \mathbb{M}_{k_n}$ be defined by
$$\theta(b) = (\theta'_n(b))_{n=1}^{\infty}.$$
By the construction, θ is unital completely positive and $\theta(J) \subset \left(\bigoplus \mathbb{M}_{k_n}\right)_{c_0}$. Regarding $\prod_{n=1}^{\infty} \mathbb{M}_{k_n}$ as a subspace of $\mathbb{B}\Big(\bigoplus_{n=1}^{\infty} \mathbb{C}^{k_n}\Big)$ canonically, we can see that $\|\overset{\vee}{\theta}(v + E \otimes_{\min} J)\| > 1$. ∎

Now, let us prove the "if" part of Proposition 2.9.

Proof. Let X be a separable operator space and assume that any complete contraction from X to $\mathbb{B}/\mathbb{K}$ has a lifting with cb-norm $\leqslant \lambda$. We give ourselves a complete contraction φ from X to a quotient C^*-algebra B/J. We may assume that B is separable. Let $E \subset X$ be any finite dimensional operator subspace and let $\varepsilon > 0$ be arbitrary. Let $u \in E^* \otimes (B/J)$ be the element corresponding to $\varphi|E$. Then, for any complete contraction $\theta : B \to \mathbb{B}$ with $\theta(J) \subset \mathbb{K}$, the element $(\mathrm{id} \otimes \dot{\theta})(u) \in E^* \otimes \mathbb{B}/\mathbb{K}$ corresponds to the complete contraction $(\dot{\theta} \circ \varphi)|E$. By the assumption, there exists a lifting of $(\dot{\theta} \circ \varphi)|E$ with cb-norm $\leqslant \lambda$. This means that $\|\overset{\vee}{\theta}(u)\|_{(E^* \otimes_{\min} \mathbb{B})/(E^* \otimes_{\min} \mathbb{K})} \leqslant \lambda$. Thus, by Lemma 2.10, we have $\|u\|_{(E^* \otimes_{\min} B)/(E^* \otimes_{\min} J)} \leqslant \lambda$. This means that there is a lifting of $\varphi|E$ with cb-norm $\leqslant \lambda + \varepsilon$ and we are done. ∎

Remark 2.11. It can be seen from the proof that we may replace $\mathbb{B}$ by $\prod_{n=1}^{\infty} \mathbb{M}_n$ and $\mathbb{K}$ by $\Big(\bigoplus_{n=1}^{\infty} \mathbb{M}_n\Big)_{c_0}$ in the statements in Proposition 2.9 and in Lemma 2.10. Moreover, a separable unital C^*-algebra A has the LLP if and only if every unital completely positive map from A into the Calkin algebra has a unital completely positive lifting. (Apply the above proof to a unital surjective $*$-homomorphism from $C^*(\mathbb{F}_\infty)$ onto A.)

For every Banach space X, the minimal operator space structure is induced by an isometric embedding of X into a commutative C^*-algebra C. We denote the resulting operator space by $\mathrm{MIN}(X)$. It is easy to see that $\mathrm{MIN}(X)$ does not depend on the choice of C and that $\mathrm{ex}(\mathrm{MIN}(X)) = 1$. For every Banach space X, the maximal operator space structure $\mathrm{MAX}(X)$ on X is defined so that any contraction from $\mathrm{MAX}(X)$ into any operator space is completely contractive. We have completely isometric identifications $\mathrm{MIN}(X)^* = \mathrm{MAX}(X^*)$ and $\mathrm{MAX}(X)^* = \mathrm{MIN}(X^*)$ and therefore $\mathrm{MAX}(X)^{**} = \mathrm{MAX}(X^{**})$. See [2], [3], [14] for details.

We list some examples. Similar results have already appeared in [10].

THEOREM 2.12. *The following is a list of spaces with the* OLP.

(i) *Every separable predual R_* of a von Neumann algebra R has the* 1-OLP.

(ii) *If X is a separable Banach space with the λ-bounded approximation property, then* MAX(X) *has the* λ-OLP.

(iii) *The Hardy space H_1 with the operator space structure induced by $H_1 \subset L_1(\mathbb{T})$ has the* 1-OLP.

(iv) *A finite dimensional operator space E has* λ-OLP *if and only if* $\mathrm{ex}(E^*) \leqslant \lambda$.

Proof. (i) Since $R \otimes_{\min} B \to R \otimes_{\min} (B/J)$ is a $*$-homomorphism with dense range, it is a complete metric surjection. It follows that all finite rank complete contractions from R_* to B/J have completely contractive liftings from R_* to B. But, by Lemma 1.6, these maps are point-norm dense in the set of all complete contractions, hence (i) follows from Lemma 2.3.

(ii) We only need to show that MAX(X) has the λ-OLLP (cf. Corollary 2.4). Let $E \subset X$ be a finite dimensional subspace and let $\varepsilon > 0$ be arbitrary. By the assumption, there is a finite rank map ρ on X with $\|\rho\|\lambda + \varepsilon$ such that $\rho|E = \mathrm{id}_E$. Then, we have $\|E \hookrightarrow \mathrm{MAX}(\rho(X))\|_{\mathrm{cb}} \leqslant \|\rho\|$ and $\mathrm{ex}(\mathrm{MAX}(\rho(X))^*) = 1$. Hence MAX($X$) has the λ-OLLP.

(iii) This follows from Lemma 4.1 in [10].

(iv) This has been proved already in remarks preceding Theorem 2.5. ∎

REMARK 2.13. In particular, the above spaces can be embedded into $C^*(\mathbb{F}_\infty)$ completely isomorphically.

There is an operator space which has the OLLP but fails to have the OLP. For instance, $\mathrm{MAX}(\ell_\infty/c_0)$ is such an example. Indeed, since ℓ_∞/c_0 has the metric approximation property, $\mathrm{MAX}(\ell_\infty/c_0)$ has the 1-OLLP, but it is well known that there is no bounded linear lifting from ℓ_∞/c_0 to ℓ_∞. We do not know if the OLLP and the OLP are equivalent for separable operator spaces.

Generally, the predual R_* of a von Neumann algebra R has the 1-OLLP and a maximal operator space MAX(X) with the λ-bounded approximation property has the λ-OLLP. The proof is similar. An operator space which contains a completely isometric copy of ℓ_∞^3 cannot have the 1-OLLP. This follows from the injectivity of ℓ_∞^3 and Proposition 3.2 in [7].

PROPOSITION 2.14. (i) *If $\{X_i\}_{i \in I}$ is a family of operator spaces with the* λ-OLLP, *then $\ell_1(I; X_i)$ (see* [18]*) has the* λ-OLLP.

(ii) *If X has the* λ-OLLP *and the map $q : X \to Y$ is a $\mathbb{B}$-metric surjection, i.e., $q \otimes \mathrm{id}_{\mathbb{B}} : X \otimes_{\min} \mathbb{B} \to Y \otimes_{\min} \mathbb{B}$ is a metric surjection, then Y has the* λ-OLLP.

(iii) *If X has the* λ-OLLP *and Y is a subspace of X such that there is a complete contraction $\varphi : X \to Y^{**}$ with $\varphi|Y = \iota_Y$, then Y has the* λ-OLLP.

Proof. (i) Easy.

(ii) Since completely bounded maps tensorize the δ-tensor product, this follows from Theorem 2.5 (ii).

(iii) Since $X \otimes_\delta \mathbb{B} \subset X^{**} \otimes_\delta \mathbb{B}$ canonically, this follows from Theorem 2.5 (ii). (Note that the residual finiteness of universal C^*-algebras gives us the canonical inclusions $C^*\langle X\rangle \subset C^*\langle X^{**}\rangle \subset C^*\langle X\rangle^{**}$.) ∎

Remark 2.15. If X is a separable operator space with the OLP and R_* is the separable predual of a von Neumann algebra R, then the projective tensor product (in the sense of [6]) $X \widehat{\otimes} R_*$ has the OLP.

If E is a finite dimensional subspace of an operator space X, then the quotient map from X onto X/E is automatically a $\mathbb{B}$-metric surjection. Hence, if moreover X has the 1-OLLP then X/E has the 1-OLLP. This argument gives us an example below. When the author asked E. Kirchberg whether the general inductive limit is continuous with respect to the minimal tensor product or not, he kindly showed a counterexample. The example below is another one.

Example 2.16. There exists a separable C^*-algebra A and an increasing sequence of ideals I_n of A such that A/I_n has the LP for all n, but A/I both fails the LLP and the WEP, where $I = \overline{\bigcup I_n}$.

Let us construct such an example. Let E be a finite dimensional subspace of $C^*(\mathbb{F}_\infty)$ such that $\mathrm{ex}(E) > 1$ (cf. [16]). Fix a complete metric surjection from S_1 onto E^* and let N be the kernel of this map. Take an increasing sequence of finite dimensional subspaces $N_n \subset N$ such that $\overline{\bigcup N_n} = N$. By the above argument, S_1/N_n has the 1-OLLP. We claim that S_1/N_n has also the complete metric approximation property. Indeed, let F be any finite dimensional subspace of S_1/N_n and let $\varepsilon > 0$ be arbitrary. Then, $F + N_n$ is a finite dimensional subspace of S_1 and there is a finite rank map φ on S_1 with $\|\varphi\|_{\mathrm{cb}} < 1 + \varepsilon$ such that $\varphi|F + N_n = \mathrm{id}|F + N_n$. This φ induces a finite rank map $\widetilde{\varphi}$ on S_1/N_n with $\|\widetilde{\varphi}\|_{\mathrm{cb}} < 1 + \varepsilon$ such that $\widetilde{\varphi}|F = \mathrm{id}_F$. This proves that S_1/N_n has the complete metric approximation property. Thus, by Corollary 2.4, S_1/N_n has the 1-OLP. Let $A = C^*\langle S_1\rangle$ and let $I_n = \ker(C^*\langle S_1\rangle \to C^*\langle S_1/N_n\rangle)$. Then $A/I = C^*\langle E^*\rangle$. Since $\mathrm{ex}(E) > 1$, E^* fails the 1-OLLP, hence A/I fails the LLP. Finally, suppose that A/I has the WEP. Then, by Proposition 1.1 in [11], we have $(A/I) \otimes_{\max} C^*(\mathbb{F}_\infty) = (A/I) \otimes_{\min} C^*(\mathbb{F}_\infty)$, hence by the definition of the δ-norm, we have $E^* \otimes_\delta C^*(\mathbb{F}_\infty) = E^* \otimes_{\min} C^*(\mathbb{F}_\infty)$. Thus, applying Corollary 6.3.5 in [18] to the inclusion map of E into $C^*(\mathbb{F}_\infty)$, we have that $\mathrm{ex}(E) = 1$. This contradicts the choice of E.

3. MAXIMAL OPERATOR SPACES

In this section, we will prove

Proposition 3.1. *The following conjectures are equivalent:*

(C1) *Every maximal operator space has the* OLLP.

(C1′) *The operator space* MAX(A) *has the* OLLP *for every separable* C^*-*algebra* A.

(C2) *Every maximal operator space is locally reflexive.*

(C2′) *Every separable maximal operator space with the metric approximation property is locally reflexive.*

(C3) *The Banach space* $\mathbb{B}$ *is extendibly locally reflexive.*

The conjecture (C2) was raised by T. Oikhberg. Following H. Rosenthal, a Banach space X is called extendibly locally reflexive if there is $\lambda > 0$ so that, for any finite dimensional subspaces $E \subset X^{**}$, $F \subset X^*$ and any $\varepsilon > 0$, there exists a map $\varphi : X^{**} \to X^{**}$ with norm $< \lambda+\varepsilon$ such that $\varphi(E) \subset X$ and $\langle \varphi(x), f\rangle = \langle x, f\rangle$ for all $x \in E$, $f \in F$. In case X is an operator space, we say that X is operator extendibly locally reflexive if one can find such φ's with cb-norm $< \lambda + \varepsilon$. For a Banach space X, it is not hard to see that X is extendibly locally reflexive if and only if MAX(X) is operator extendibly locally reflexive. H. Rosenthal conjectured that (C3) would be false.

To prove the proposition, we need several lemmas.

Lemma 3.2. *Let X be a Banach space. Then* MAX(X) *has the* λ-OLLP *if and only if for every finite dimensional subspace* $E \subset \mathrm{MAX}(X)$ *and* $\varepsilon > 0$ *there exists a finite dimensional subspace* $F \subset X$ *containing* E *such that* $\|E \hookrightarrow \mathrm{MAX}(F)\|_{\mathrm{cb}} < \lambda + \varepsilon$.

Proof. Since $\mathrm{MAX}(F)^* = \mathrm{MIN}(F^*)$ is 1-exact, the 'if' part follows from (i) $\Rightarrow$ (iii) in Theorem 2.5. Now, let us prove the 'only if' part. Let X be a Banach space such that MAX(X) has the λ-OLLP and let $E \subset \mathrm{MAX}(X)$ be a finite dimensional subspace. Then, there exists a finite dimensional maximal operator space G (combine Theorem 2.1 in [14] and Theorem 6.3.1 in [18]) and two maps $\beta : E \to G$ and $\alpha : G \to \mathrm{MAX}(X)$ such that $\alpha\circ\beta = \mathrm{id}_E$ and $\|\alpha\|_{\mathrm{cb}} < \lambda+\varepsilon$, $\|\beta\|_{\mathrm{cb}} = 1$. Let $F = \alpha(G)$. Since G is maximal, α can be regarded as a complete contraction $\widetilde{\alpha} : G \to \mathrm{MAX}(F)$. Hence

$$\|E \hookrightarrow MAX(F)\|_{\mathrm{cb}} \leqslant \|\widetilde{\alpha}\|_{\mathrm{cb}}\|\beta\|_{\mathrm{cb}} < \lambda + \varepsilon. \quad ∎$$

Lemma 3.3. *Every separable subspace of a maximal operator space is embeddable into* MAX($\mathbb{B}$) *completely isometrically.*

Proof. Let $Y \subset \mathrm{MAX}(X)$ be a separable subspace. Take a completely isometric embedding $i : Y \to \mathbb{B}$ and extend it to a complete contraction $j : \mathrm{MAX}(X) \to \mathbb{B}$. Then, we may regard j as a complete contraction $\widetilde{j} : \mathrm{MAX}(X) \to \mathrm{MAX}(\mathbb{B})$. Consequently, $\widetilde{i} = \widetilde{j}|Y$ is a completely isometric embedding of Y into MAX($\mathbb{B}$). ∎

LEMMA 3.4. *Let X be a Banach space and $E \subset X$ be a separable subspace. Then there exists a separable subspace $X_0 \subset X$ containing E such that the inclusion* $\mathrm{MAX}(X_0) \hookrightarrow \mathrm{MAX}(X)$ *is completely isometric.*

Proof. This follows from Corollary 2.5 in [14]. It is also well known that for $E \subset X$ there exist a separable subspace $X_0 \subset X$ containing E and a contraction $T : X \to X_0^{**}$ such that $T|X_0$ is the canonical inclusion of X_0 into X_0^{**}. ∎

Now, we prove Proposition 3.1. We are indebted to T. Oikhberg for the usage of injectivity in the proof of (C2) ⇒ (C3).

Proof. (C1′) ⇒ (C1). Suppose that $\mathrm{MAX}(A)$ has the OLLP for every separable C^*-algebra A. Then, it can be seen that there exists a universal constant $\lambda > 0$ such that $\mathrm{MAX}(A)$ has the λ-OLLP for all A. Let X be a Banach space and let E be a finite dimensional subspace of $\mathrm{MAX}(X)$. By Lemma 3.4, there exists a separable subspace X_0 of $\mathrm{MAX}(X)$ containing E which is a maximal operator space. Let φ be a complete contraction from $\mathrm{MAX}(X)$ to a quotient C^*-algebra B/J and let $\varepsilon > 0$. Then, there exists a separable C^*-subalgebra B_0 of B such that $\varphi(X_0) \subset \pi(B_0)$. Since X_0 is maximal, we may regard φ as a complete contraction $\widetilde{\varphi}$ from X_0 into $\mathrm{MAX}(\pi(B_0))$. Since $\mathrm{MAX}(\pi(B_0))$ has the λ-OLLP, there exists a lifting $\psi : \widetilde{\varphi}(E) \to B_0$ with $\|\psi\|_{\mathrm{cb}} < \lambda + \varepsilon$ such that $\pi \circ \psi$ is the formal identity map from $\widetilde{\varphi}(E)$ onto $\varphi(E)$. Thus, $\psi \circ \widetilde{\varphi}|E$ is a lifting of $\varphi|E$ with cb-norm $< \lambda + \varepsilon$. Since E, φ and ε were arbitrary, this means that $\mathrm{MAX}(X)$ has the λ-OLLP.

(C1) ⇒ (C2). Since $\mathrm{MAX}(X^{**}) = \mathrm{MAX}(X)^{**}$ has the OLLP, $\mathrm{MAX}(X)$ is locally reflexive by Corollary 2.7.

(C2) ⇒ (C2′). Obvious.

(C2′) ⇒ (C1′). Let X be a separable Banach space. Take an increasing sequence of finite dimensional subspaces E_n of X such that $\bigcup E_n$ is dense in X. Let $Y = \big(\bigoplus E_n\big)_{\ell_1}$. By the assumption, $\mathrm{MAX}(Y)$ is λ-locally reflexive for some $\lambda > 0$. We define a metric surjection q from Y onto X by $q((x_n)_n) = \sum x_n$. Then, there exists a contractive map $r : X^{**} \to Y^{**}$ with $q^{**} \circ r = \mathrm{id}_{X^{**}}$ (cf. Proposition 1 in [8]). We note that r is completely contractive as a map from $\mathrm{MAX}(X^{**})$ to $\mathrm{MAX}(Y^{**})$. To see that $\mathrm{MAX}(X)$ has the λ-OLLP, we give ourselves a finite dimensional subspace E of $\mathrm{MAX}(X)$ and $\varepsilon > 0$. Let $j_E : E \hookrightarrow X$ be the inclusion map of E into X and let $\iota : \mathrm{MAX}(X) \to \mathrm{MAX}(X^{**})$ be the canonical inclusion. Since $\mathrm{MAX}(Y)$ is locally reflexive, there exists a net $\{\psi_i\}_i$ of maps from E to $\mathrm{MAX}(Y)$ with cb-norm $\leqslant \lambda$ which converges to $r \circ \iota \circ j_E : E \to \mathrm{MAX}(Y^{**})$ in the point-weak* topology. Let $\varphi_i = q \circ \psi_i : E \to \mathrm{MAX}(X)$. Then, it can be seen that the net $\{\varphi_i\}_i$ converges to j_E in the point-weak topology. Thus, we have proved that j_E is in the point-weak closure of the set of maps from E to $\mathrm{MAX}(X)$ which have liftings with cb-norm $\leqslant \lambda$. By the standard convexity argument and by the small perturbation argument, there exists a map $\psi : E \to \mathrm{MAX}(Y)$ with cb-norm $< \lambda + \varepsilon$ such that $q \circ \psi = j_E$. Since Y has the metric approximation property, there exists a finite dimensional subspace F of Y containing $\psi(E)$ such that $\|\psi(E) \hookrightarrow \mathrm{MAX}(F)\|_{\mathrm{cb}} < 1 + \varepsilon$. Hence, $\|E \hookrightarrow \mathrm{MAX}(q(F))\|_{\mathrm{cb}} < (1+\varepsilon)(\lambda+\varepsilon)$ as desired. We conclude by Lemma 3.2.

(C2) ⇒ (C3). Let E be a finite dimensional subspace of $\mathrm{MAX}(\mathbb{B}^{**})$ and let F be a finite dimensional subspace of $\mathbb{B}^*$. By the assumption, $\mathrm{MAX}(\mathbb{B})$ is λ-locally reflexive for some $\lambda > 0$. Hence, there exists a map $\varphi : E \to \mathrm{MAX}(\mathbb{B})$ with cb-norm $< \lambda + \varepsilon$ such that $\langle \varphi(x), f\rangle = \langle x, f\rangle$ for all $x \in E$ and $f \in F$. We can regard

φ as a map from E to $\mathbb{B}$ with cb-norm $< \lambda + \varepsilon$ and extend it to a map $\overline{\varphi}$ from MAX($\mathbb{B}^{**}$) to $\mathbb{B}$ with cb-norm $< \lambda + \varepsilon$. This proves that $\mathbb{B}$ is λ-extendibly locally reflexive.

(C3) $\Rightarrow$ (C2′). By the assumption, MAX($\mathbb{B}$) is locally reflexive. Since any separable maximal operator space can be embedded into MAX($\mathbb{B}$) completely isometrically and local reflexivity passes to subspaces, we are done. ∎

REMARK 3.5. A negative answer to the above conjectures would provide us with an example of a separable C^*-algebra containing a non-complemented closed two-sided ideal ($*$-isomorphic to $\mathbb{K}$). A positive answer would imply that all the QWEP algebras of Kirchberg ([11]) are extendibly locally reflexive.

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