J. OPERATOR THEORY
46(2001), 605–618

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COMPOSITION OPERATORS BETWEEN NEVANLINNA CLASSES AND BERGMAN SPACES WITH WEIGHTS

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Communicated by Florian-Horia Vasilescu

Abstract. We investigate composition operators between spaces of analytic functions on the unit disk Δ in the complex plane. The spaces we consider are the weighted Nevanlinna class $\mathcal{N}_\alpha$, which consists of all analytic functions f on Δ such that $\int\limits_\Delta \log^+ |f(z)|(1-|z|^2)^\alpha \,\mathrm{d}x\,\mathrm{d}y < \infty$, and the corresponding weighted Bergman spaces $\mathcal{A}^p_\alpha$, $-1 < \alpha < \infty$, $0 < p < \infty$. Let X be any of the spaces $\mathcal{A}^p_\alpha$, $\mathcal{N}_\alpha$ and Y any of the spaces $\mathcal{A}^q_\beta$, $\mathcal{N}_\beta$, $\beta > -1$, $0 < q < \infty$. We characterize, in function theoretic terms, when the composition operator $C_\varphi : f \mapsto f \circ \varphi$ induced by an analytic function $\varphi : \Delta \to \Delta$ defines an operator $X \to Y$ which is continuous, respectively compact, respectively order bounded.

Keywords: *Composition operators, continuity, compactness, order boundedness, weighted Nevanlinna classes, weighted Bergman spaces.*

MSC (2000): Primary 46E15, 47B38; Secondary 30D35.

1. STATEMENT OF THE RESULTS

Let Δ be the unit disk $\{z \in \mathbb{C} : |z| < 1\}$ in the complex plane, and let $\mathcal{H}(\Delta)$ be the space of all analytic functions $\Delta \to \mathbb{C}$. Any analytic map $\varphi : \Delta \to \Delta$ gives rise to an operator $C_\varphi : \mathcal{H}(\Delta) \to \mathcal{H}(\Delta)$ defined by $C_\varphi(f) := f \circ \varphi$, the *composition operator* induced by φ.

Suppose we are given two linear subspaces X and Y of $\mathcal{H}(\Delta)$, endowed with suitable topologies. One of the central problems on composition operators is to know when C_φ maps X into Y and in fact to compare function theoretic properties of φ and operator theoretic properties of $C_\varphi : X \to Y$.

Such problems are addressed here for weighted Nevanlinna classes and weighted Bergman spaces with respect to boundedness, compactness and order boundedness of the operator. As for the latter concept; see e.g. ([6]). Its importance is due to close relations to absolutely summing operators and their relatives and, accordingly, to a variety of factorization properties. Special cases of the problems to be discussed in the sequel have recently been studied by various authors, in particular within the setting of Hardy spaces and the classical Nevanlinna class $\mathcal{N}$ as well as of standard Bergman spaces and of the area version of $\mathcal{N}$; see [19], [20], [8], [3], [4], [11], [22], for example.

Let $-1 < \alpha < \infty$. We use the standard weighted area measure m_α on Δ and work with the corresponding weighted Bergman spaces $\mathcal{A}^p_\alpha$, $(0 < p < \infty)$ and the weighted Nevanlinna class $\mathcal{N}_\alpha$ which can be thought of as a limit case of the $\mathcal{A}^p_\alpha$ as $p \to 0$. Definitions will be given in the next section. Here we only explain what is needed to understand the statements of the results.

In this paper, *arcs* in the unit circle $\partial\Delta$ will be sets of the form $I = \{z \in \partial\Delta : \theta_1 \leqslant \arg z < \theta_2\}$ where $\theta_1, \theta_2 \in [0, 2\pi)$ and $\theta_1 < \theta_2$. Normalized length of an arc I will be denoted by $|I|$, so $|I| = (2\pi)^{-1} \int_I |\mathrm{d}z|$. The *Carleson box* based on an arc I is the set

$$S(I) := \bigl\{z \in \Delta : 1 - |I| \leqslant |z| < 1,\ z/|z| \in I\bigr\}.$$

Let s be any positive number. A Borel measure μ on Δ is called an *s-Carleson measure* if $\mu(S(I)) = \mathrm{O}(|I|^s)$ on Carleson boxes $S(I)$. μ is said to be a *compact s-Carleson measure* if even $\mu(S(I)) = \mathrm{o}(|I|^s)$ is true. Here $\mathrm{O}(\cdot)$ and $\mathrm{o}(\cdot)$ are the usual Landau symbols.

We present our results in four theorems. We start with continuity and compactness.

THEOREM 1.1. *Let $\alpha, \beta \in (-1, \infty)$ and $\varphi : \Delta \to \Delta$ analytic.*

(i) *$C_\varphi : \mathcal{N}_\alpha \to \mathcal{N}_\beta$ exists as a continuous operator if and only if $m_{\beta,\varphi}$ is a $(\alpha + 2)$-Carleson measure: $m_{\beta,\varphi}(S(I)) = \mathrm{O}(|I|^{\alpha+2})$.*

(ii) *$C_\varphi : \mathcal{N}_\alpha \to \mathcal{N}_\beta$ exists as a compact operator if and only if $m_{\beta,\varphi}$ is a compact $(\alpha + 2)$-Carleson measure: $m_{\beta,\varphi}(S(I)) = \mathrm{o}(|I|^{\alpha+2})$.*

Here $m_{\beta,\varphi}$ is the image measure $m_\beta \circ \varphi^{-1}$ on the Borel sets of Δ.

Order boundedness can be characterized as follows:

THEOREM 1.2. *Let $\alpha, \beta \in (-1, \infty)$ and $\varphi : \Delta \to \Delta$ analytic. $C_\varphi : \mathcal{N}_\alpha \to \mathcal{N}_\beta$ exists as an order bounded operator if and only if $(1 - |\varphi|^2)^{-(\alpha+2)} \in L^1(m_\beta)$.*

Theorems 1.1 and 1.2 complement nicely known characterizations for composition operators between weighted Bergman spaces (see [8], Propositions 2, 3, and 6):

(A) *$C_\varphi : \mathcal{A}^p_\alpha \to \mathcal{A}^q_\beta$ exists as a continuous operator if and only if $m_{\beta,\varphi}$ is a $(\alpha + 2)q/p$-Carleson measure: $m_{\beta,\varphi}(S(I)) = \mathrm{O}(|I|^{(\alpha+2)q/p})$.*

(B) *$C_\varphi : \mathcal{A}^p_\alpha \to \mathcal{A}^q_\beta$ exists as a compact operator if and only if $m_{\beta,\varphi}$ is a compact $(\alpha + 2)q/p$-Carleson measure: $m_{\beta,\varphi}(S(I)) = \mathrm{o}(|I|^{(\alpha+2)q/p})$.*

(C) *$C_\varphi : \mathcal{A}^p_\alpha \to \mathcal{A}^q_\beta$ exists as order bounded operator if and only if $(1 - |\varphi|^2)^{-(\alpha+2)q/p} \in L^1(m_\beta)$.*

Here $0 < p \leqslant q < \infty$. Notice that, for any choice of $-1 < \alpha, \beta < \infty$ and $0 < p < \infty$, the results for $C_\varphi : \mathcal{N}_\alpha \to \mathcal{N}_\beta$ are the same as for $C_\varphi : \mathcal{A}^p_\alpha \to \mathcal{A}^p_\beta$. We refer to Chapter 5 in [6] for unexplained terminology and facts on order bounded operators and related material. We only mention that a Hilbert space operator is order bounded if and only if it is Hilbert-Schmidt.

In general, every L_p-valued order bounded operator factorizes in a canonical fashion through a space $C(K)$ of continuous functions on some compact Hausdorff space K and is in fact a p-integral operator. For composition operators $C_\varphi : \mathcal{A}^p_\alpha \to \mathcal{A}^q_\beta$ one can do better. Let X be the space of all $f \in \mathcal{H}(\Delta)$ such that $\sup_{z\in\Delta}(1-|z|^2)^{(\alpha+2)}|f(z)|$ is finite. It is well-known that X is a Banach space with respect to this expression and isomorphic to ℓ_∞, and that $\mathcal{A}^p_\alpha$ embeds continuously into X. Now $C_\varphi : \mathcal{A}^p_\alpha \to \mathcal{A}^q_\beta$ is order bounded iff C_φ actually maps X into $\mathcal{A}^q_\beta$ and it is then automatically order bounded. This can be proved either by modifying the argument given in [7], Theorem 7.1, or by adaption of the lacunary series argument from [1], Theorem 16, as in [14].

The following consequence of Theorem 1.2 may appear unexpected. Looking at formal identities $\mathcal{N}_\alpha \hookrightarrow \mathcal{N}_\beta$ as the composition operators induced by the identity map $\Delta \to \Delta$, we may state:

COROLLARY 1.3. *Let $\alpha, \beta > -1$ be given. The following are equivalent:*
(i) $\beta > \alpha + 1$.
(ii) *$\mathcal{N}_\alpha$ is a subset of $\mathcal{N}_\beta$, and the corresponding embedding is order bounded.*
(iii) *For some (and then every) $0 < p < \infty$, $\mathcal{A}^p_\alpha$ is a subset of $\mathcal{A}^p_\beta$, and the corresponding embedding is order bounded.*

Of course, the equivalence of (i) and (iii) can also be seen directly once the independence of p is known. Moreover, by atomic decomposition, each A^p_β is isomorphic to ℓ_p, so that, for example in the case $p = 1$, we are just discussing nuclearity of the canonical embedding $\mathcal{A}^1_\alpha \hookrightarrow \mathcal{A}^1_\beta$.

The case of operators from $\mathcal{A}^p_\alpha$ to $\mathcal{N}_\beta$ and from $\mathcal{N}_\alpha$ to $\mathcal{A}^q_\beta$ appears to be interesting enough to be treated separately.

THEOREM 1.4. *No matter how we choose $-1 < \alpha,\ \beta < \infty$, $0 < p < \infty$, and the analytic function $\varphi : \Delta \to \Delta$, $C_\varphi : \mathcal{A}^p_\alpha \to \mathcal{N}_\beta$ is always well-defined and continuous; it is even order bounded and compact.*

Again, this applies in particular to embeddings $\mathcal{A}^p_\alpha \hookrightarrow \mathcal{N}_\beta$.

The other result reads as follows:

THEOREM 1.5. *Let $\varphi : \Delta \to \Delta$ be analytic, $-1 < \alpha,\ \beta < \infty$ and $0 < q < \infty$. Then C_φ exists as a continuous operator $\mathcal{N}_\alpha \to \mathcal{A}^q_\beta$ if and only if*

$$\exp \frac{1}{(1-|\varphi|^2)^{\alpha+2}} \in \bigcap_{r>0} L^r(m_\beta),$$

and in this case, the operator is compact and order bounded.

Note that the condition is independent of q, so that in the above situation C_φ actually maps $\mathcal{N}_\alpha$ into $\bigcap_{q>0} A^q_\beta$.

The integrability condition in Theorem 1.5 is equivalent to requiring convergence of $\sum_{n=1}^{\infty} \exp(c\, n^{(\alpha+2)/(\alpha+3)}) \cdot \|\varphi^n\|_{1,\beta}$ for some $c > 0$. Using this it is possible to show that composition operators $C_\varphi : \mathcal{N}_\alpha \to \mathcal{A}^q_\beta$ enjoy strong nuclearity properties, obtained from the standard Banach space concepts in a natural manner by taking into account $\mathcal{N}_\alpha$'s structure as an F-space. We plan to return to such topics elsewhere.

The situation for the classical Nevanlinna class $\mathcal{N}$ and Hardy spaces H^p has recently been closely investigated by N. Jaoua ([11]) and J.S. Choa, H.O. Kim and J.H. Shapiro ([4]); some of the above results even extend formally to this case by just allowing α to take the value -1. We mention the following extension of a result from the latter paper. We take $\mathcal{N}_{-1}$ to be the Smirnov class, usually denoted $\mathcal{N}^+$.

COROLLARY 1.6. *For any $\alpha > -1$, every composition operator is compact as an operator $\mathcal{N} \to \mathcal{N}_\alpha$; it is order bounded if and only if $\alpha > 0$.*

In particular, $\mathcal{N} \hookrightarrow \mathcal{N}_\alpha$ is compact for $\alpha > -1$ and order bounded iff $\alpha > 0$.

Here compactness and order boundedness for operators $\mathcal{N} \to \mathcal{N}_\alpha$ are defined by straightforward generalization of the concepts used before (see [4] and [11]). However, we can also rely on the old definition by observing that the operators in question contain compact, respectively order bounded, factors $\mathcal{N}_{\alpha'} \to \mathcal{N}_\alpha$ for suitably chosen $-1 < \alpha' < \alpha$.

The proof is easy modulo known results. One of the main results in [4] is that a composition operator on $\mathcal{N}$ is compact if and only if it takes its values in $\mathcal{N}^+$. Corollary 1.6 can be deduced from this as follows. It is clear that $\mathcal{N}$ is a linear subspace of $\mathcal{N}_\alpha$. Moreover, as was shown by T. Domenig ([7], Theorem 8.4), there is an analytic map $\psi : \Delta \to \Delta$ such that, simultaneously for each $0 < p < \infty$, C_ψ defines an isomorphic embedding of $\mathcal{A}^p_\alpha$ into H^p. An examination of the argument reveals that C_ψ also provides an isomorphic embedding of $\mathcal{N}_\alpha$ into $\mathcal{N}^+$. Its restriction to $\mathcal{N}$ is compact, by [4], and so $\mathcal{N} \hookrightarrow \mathcal{N}_\alpha$ is compact as well. The order boundedness part follows from Corollary 1.3.

We refer to the next section for definitions and terminology, and for the basic material on weighted Nevanlinna classes $\mathcal{N}_\alpha$ and related spaces. The proofs of the Theorems 1.1, 1.2, 1.4 and 1.5 will then be given in the subsequent sections.

2. BACKGROUND

To keep our notation simple, we shall frequently denote positive constants just by C and c allowing them to change their meaning from line to line. Also, given two families $x = (x(\omega))_{\omega\in\Omega}$ and $y = (y(\omega))_{\omega\in\Omega}$ of non-negative real numbers (or functions), we write $x \asymp y$ if (there exist constants $c, C > 0$ such that) $cx(\omega) \leqslant y(\omega) \leqslant Cx(\omega)$ for all $\omega \in \Omega$.

Let Δ be the unit disk $\{z \in \mathbb{C} : |z| < 1\}$ in the complex plane, and let m be normalized area measure on Δ; so $\mathrm{d}m(x + \mathrm{i}y) = \pi^{-1}\,\mathrm{d}x\,\mathrm{d}y$. It is well-known that for each $-1 < \alpha < \infty$,

$$\mathrm{d}m_\alpha(z) := (\alpha + 1)\,(1 - |z|^2)^\alpha\,\mathrm{d}m(z),$$

is a probability measure on (the Borel algebra of) Δ. For $0 < p < \infty$, the canonical norm (a p-norm if $0 < p < 1$) on the corresponding Lebesgue space $L^p(m_\alpha)$ will be denoted by $\|\cdot\|_{p,\alpha}$; so that $\|f\|_{p,\alpha} = \big(\int_\Delta |f|^p\,\mathrm{d}m_\alpha\big)^{1/p}$. As a 'limit case' of the

$L_p(m_\alpha)$, we introduce the space $L_{\log^+}(m_\alpha)$ of all (m_α-a.e. equivalence classes of) measurable functions f on Δ satisfying

$$T_\alpha(f) := \int_\Delta \log^+ |f| \, \mathrm{d}m_\alpha < \infty.$$

As usual, $\log^+ x$ is $\log x$ if $x > 1$ and 0 if $0 \leqslant x \leqslant 1$. From $p \log^+ x \leqslant x^p$ for all $p \in (0, \infty)$ and $x \in [0, \infty)$ we get

$$p\, T_\alpha(f) \leqslant \|f\|_{p,\alpha}^p$$

if $f \in L^p(m_\alpha)$. It follows that $L_{\log^+}(m_\alpha)$ contains each of the spaces $L^p(m_\alpha)$, $0 < p < \infty$.

Since $\log^+ x \leqslant \log(1 + x) \leqslant \log 2 + \log^+ x$ for $x \geqslant 0$, a measurable function f on Δ belongs to $L_{\log^+}(m_\alpha)$ if and only if

$$|||f|||_\alpha := \int_\Delta \log(1 + |f|) \, \mathrm{d}m_\alpha$$

is finite. We write $|||f|||_\alpha = \infty$ if $f \notin L_{\log^+}(m_\alpha)$. Obviously,

$$\max\{|||f + g|||_\alpha,\, |||fg|||_\alpha\} \leqslant |||f|||_\alpha + |||g|||_\alpha \tag{2.1}$$

for all $f, g \in L_{\log^+}(m_\alpha)$. Consequently, $L_{\log^+}(m_\alpha)$ is not only a vector space but even an algebra. It also follows from (2.1) that, by setting

$$d_\alpha(f, g) := |||f - g|||_\alpha$$

for $f, g \in L_{\log^+}(m_\alpha)$, we obtain a translation invariant metric on $L_{\log^+}(m_\alpha)$. More is true:

PROPOSITION 2.1. *For any $\alpha \in (-1, \infty)$, $|||\cdot|||_\alpha$ is an F-norm under which $L_{\log^+}(m_\alpha)$ is an F-space.*

Moreover, d_α-convergence implies convergence in measure, and the canonical embedding $L^p(m_\alpha) \hookrightarrow L_{\log^+}(m_\alpha)$ is continuous ($0 < p < \infty$).

Here an *F-space* is just a complete metrizable topological vector space. See [12] or [15] for the definition and properties of F-norms.

Proof. We omit the routine verification of the F-norm properties. It is also easy to see that convergence in measure is weaker than convergence with respect to d_α.

To prove completeness, let (f_n) be a Cauchy sequence in $L_{\log^+}(m_\alpha)$ and let f be its limit with respect to convergence in measure. It follows that every subsequence of (f_n) admits a further subsequence (g_k) which converges m_α-a.e. to f. By Fatou's lemma, $\int_\Delta \log(1 + |g_k - f|) \, \mathrm{d}m_\alpha$ becomes as small as we please provided we choose k large enough. It follows that f belongs to $L_{\log^+}(m_\alpha)$ and is the d_α-limit of (g_k), hence of (f_n).

That the embedding $L^p(m_\alpha) \hookrightarrow L_{\log^+}(m_\alpha)$ is continuous can be derived directly from relations stated above, or by using the closed graph theorem. ∎

$\mathcal{H}(\Delta)$ is a Fréchet space (locally convex, metrizable and complete) with respect to the compact-open topology, that is, the topology of uniform convergence on compact subsets of Δ; in fact, $\mathcal{H}(\Delta)$ is even a Fréchet algebra. By Montel's theorem, bounded sets in $\mathcal{H}(\Delta)$ are relatively compact; accordingly, bounded sequences in $\mathcal{H}(\Delta)$ admit convergent subsequences. Convergence in this space will be referred to as *local uniform* (*l.u.*) *convergence*.

Let again $-1 < \alpha < \infty$ and $0 < p < \infty$ be given. The *weighted Bergman space*

$$\mathcal{A}^p_\alpha := \mathcal{H}(\Delta) \cap L^p(m_\alpha)$$

is a closed linear subspace of the (p-)Banach space $L^p(m_\alpha)$.

The *weighted Nevanlinna class* is defined as the set

$$\mathcal{N}_\alpha := \mathcal{H}(\Delta) \cap L_{\log^+}(m_\alpha) \ ;$$

it is a linear subspace (even a subalgebra) of $\mathcal{H}(\Delta)$ which contains all the spaces $\mathcal{A}^p_\alpha$, $0 < p < \infty$. Take note of the fact that $\mathcal{N}_\alpha$ is a topological vector space with respect to the induced F-norm $|||\cdot|||_\alpha$. This is in contrast to the situation for the classical Nevanlinna class $\mathcal{N}$, which is far from being a topological vector space ([19]), but the Smirnov class $\mathcal{N}^+$ (see [9]) can be looked at as the case $\alpha = -1$ of the spaces $\mathcal{N}_\alpha$. We have:

PROPOSITION 2.2. *With respect to* $|||\cdot|||_\alpha$, $\mathcal{N}_\alpha$ *is an* F*-space whose topology is stronger than that of local uniform convergence.*

Proof. To see that convergence with respect to $|||\cdot|||_\alpha$ implies l.u. convergence, recall that if $f \in \mathcal{N}_\alpha$ then, by subharmonicity of $\log(1 + |f|)$,

$$\log(1 + |f(z)|) \leqslant \frac{M_0|||f|||_\alpha}{(1 - |z|^2)^{\alpha+2}} \tag{2.2}$$

for all $z \in \Delta$, where M_0 is a constant depending only on α. In particular, if (f_n) is a Cauchy sequence in $\mathcal{N}_\alpha$ then it converges l.u. to some $f \in \mathcal{H}(\Delta)$. As before, by Fatou's lemma, f belongs to $\mathcal{N}_\alpha$ and satisfies $\lim_n |||f - f_n|||_\alpha = 0$ — which was what we wanted. ∎

It is clear that if $-1 < \alpha < \beta < \infty$, then $\mathcal{N}_\alpha$ is contained in $\mathcal{N}_\beta$ and the embedding is continuous. To see that the containment is strict just look at the asymptotic behaviour of $|||f_w|||_\alpha$ and $|||f_w|||_\beta$ as $|w| \to 1$, where $f_w(z) = \exp\left[\left(\frac{1 - |w|^2}{(1 - \overline{w}z)^2}\right)^{\beta+2}\right]$ for $w \in \Delta$. It is equally clear that $\mathcal{N}$ embeds continuously into $\mathcal{N}_\alpha$, for every $\alpha > -1$.

Proposition 2.2 will be applied to characterize continuity and compactness of a composition operator $C_\varphi : X \to Y$ where X and Y are taken from the spaces $\mathcal{N}_\alpha$ and $\mathcal{A}^q_\alpha$, with possibly varying parameters. Subsets B in $\mathcal{N}_\alpha$ are called bounded if they are bounded for the defining linear topology. Note that a linear map $u : \mathcal{N}_\alpha \to Y$ is continuous iff it is bounded in the sense that $u(B) \subset Y$ is bounded for every bounded subset B of $\mathcal{N}_\alpha$. We say that u is compact if $u(B) \subset Y$ is relatively compact for some 0-neighbourhood $B \subset \mathcal{N}_\alpha$.

As usual, compactness of $C_\varphi : X \to Y$ can be described as follows:

Proposition 2.3. *Let $\varphi : \Delta \to \Delta$ be analytic. Then $C_\varphi : X \to Y$ is compact if and only if for every sequence (f_n) which is bounded in X and converges to* 0 *l.u., $\lim_{n\to\infty} \|C_\varphi f_n\|_Y = 0$.*

Proof. This is known for $X = \mathcal{A}^p_\alpha$, so it suffices to look at the case where X is $\mathcal{N}_\alpha$ and Y is $\mathcal{N}_\beta$ or $\mathcal{A}^q_\beta$; accordingly, $\|\cdot\|_Y$ is $|||\cdot|||_\beta$ or $\|\cdot\|_{q,\beta}$.

Suppose first that $C_\varphi : \mathcal{N}_\alpha \to Y$ is compact. Let (f_n) be a bounded sequence in $\mathcal{N}_\alpha$ which converges l.u. to 0. We argue by contradiction and assume that $\|C_\varphi f_n\|_Y$ does not converge to zero. Passing to a subsequence if necessary, we may assume that $\inf_n \|C_\varphi f_n\|_Y > 0$. Using the compactness of C_φ and passing to another subsequence if needed, we can assume that $\lim_{n\to\infty} \|g - C_\varphi f_n\|_Y = 0$ for some $g \in Y$. If $Y = \mathcal{N}_\beta$, then

$$\log\left(1 + |(g - C_\varphi f_n)(z)|\right) \leqslant \frac{M_0|||g - C_\varphi f_n|||_\beta}{(1-|z|)^{\beta+2}}$$

by (2.2), and if $Y = \mathcal{A}^q_\beta$, then

$$|(g - C_\varphi f_n)(z)| \leqslant \frac{\|g - C_\varphi f_n\|_{q,\beta}}{(1-|z|^2)^{(\beta+2)/q}}$$

for all $z \in \Delta$; see [21]. Hence $g - C_\varphi f_n \to 0$ l.u. But $f_n \to 0$ l.u. implies $C_\varphi f_n \to 0$ l.u., so that $g = 0$ and $\lim_{n\to\infty} \|C_\varphi f_n\|_Y = 0$: contradiction!

Next we show that the condition in Proposition 2.3 implies that $C_\varphi : \mathcal{N}_\alpha \to Y$ is compact. For this, let (f_n) be a bounded sequence in $\mathcal{N}_\alpha$. By Proposition 2.1, (f_n) is bounded in $\mathcal{H}(\Delta)$ and so, by Montel's theorem, admits a subsequence (f_{n_k}) which converges l.u. to some $f \in \mathcal{H}(\Delta)$. As in the proof of Proposition 2.1, f belongs to $\mathcal{N}_\alpha$, hence $\lim_{k\to\infty} \|C_\varphi(f - f_{n_k})\|_Y = 0$. The proof is complete. ∎

Let X be a quasi-Banach space and Y a subspace of a vector lattice L. We say that an operator $u : X \to Y$ is *order bounded* if it maps the unit ball B_X of X into an order interval of L: for some $g \geqslant 0$ in L we have $|Tf| \leqslant g$ for all $f \in B_X$. Within the setting of composition operators on Hardy spaces and weighted Bergman spaces, this concept was investigated in [10], [13] and [8]. We take $L = L^q(m_\alpha)$ if Y is A^q_β, and $L = L_{\log^+}(m_\beta)$ if Y is $\mathcal{N}_\beta$. To extend this definition to $X = \mathcal{N}_\alpha$ we follow [11] and say that $u : X \to Y$ is order bounded if every ball $B_X(0,s) = \{x \in X : d(x,0) \leqslant s\}$ ($s > 0$, d the given metric of X) is mapped into an order interval: $M(u,s) := \sup_{x\in B_X(0,s)} |u(x)|$ exists in L for every $s > 0$.

3. PROOF OF THEOREM 1.1

We keep the preceding notations. Note that the measure $m_{\beta,\varphi} = m_\beta \circ \varphi^{-1}$ lives on $\varphi(\Delta)$ and satisfies

$$\int_\Delta \log(1+|f\circ\varphi|)\,\mathrm{d}m_\beta = \int_\Delta \log(1+|f|)\,\mathrm{d}m_{\beta,\varphi} \tag{3.1}$$

for all $f \in \mathcal{N}_\beta$.

(i) Assume first of all that $C_\varphi : \mathcal{N}_\alpha \to \mathcal{N}_\beta$ is continuous. In order to show that $m_{\beta,\varphi}$ is an $(\alpha+2)$-Carleson measure, fix $\theta \in [0, 2\pi)$, $h \in (0,1]$, put $w = (1-h)\mathrm{e}^{\mathrm{i}\theta}$ and consider the test function

$$f_w(z) = \exp\left[\left(\frac{1-|w|^2}{(1-\overline{w}z)^2}\right)^{\alpha+2}\right].$$

Since $\log(1+x) \leqslant 1 + \log^+ x$ for $x \geqslant 0$,

$$|||f_w|||_\alpha \leqslant 1 + \int_\Delta \log^+|f_w|\,\mathrm{d}m_\alpha \leqslant 1 + \int_\Delta \left|\frac{1-|w|^2}{(1-\overline{w}z)^2}\right|^{\alpha+2} \mathrm{d}m_\alpha(z) \leqslant K,$$

where K is a constant depending only on α. Here we have used Lemma 4.2.2 of [23]. If I is the arc centered at $\mathrm{e}^{\mathrm{i}\theta}$ of sufficiently small (normalized) length h, then $|1-\overline{w}z|^{-4(\alpha+2)} \geqslant c\,h^{-4(\alpha+2)}$ and $\mathrm{Re}\big[(1-w\overline{z})^{2(\alpha+2)}\big] \geqslant c\,h^{2(\alpha+2)}$ for $z \in S(I)$, hence

$$\begin{aligned}\log^+|f_w(z)| &= \log^+\left|\exp\frac{(1-|w|^2)^{\alpha+2}\,(1-w\overline{z})^{2(\alpha+2)}}{|1-\overline{w}z|^{4(\alpha+2)}}\right| \\ &= \frac{(1-|w|^2)^{\alpha+2}\,\mathrm{Re}\big[(1-w\overline{z})^{2(\alpha+2)}\big]}{|1-\overline{w}z|^{4(\alpha+2)}} \geqslant \frac{c^2}{h^{\alpha+2}}\end{aligned}$$

and so, by (3.1) and since $\log^+ x \leqslant \log(1+x)$ on $[0,\infty)$,

$$|||C_\varphi f_w|||_\beta \geqslant \int_{S(I)} \log^+|f_w|\,\mathrm{d}m_{\beta,\varphi} \geqslant \frac{c^2}{h^{\alpha+2}} \cdot m_{\beta,\varphi}(S(I)).$$

This proves that $m_{\beta,\varphi}$ is an $(\alpha+2)$-Carleson measure.

Suppose conversely that $m_{\beta,\varphi}$ is an $(\alpha+2)$-Carleson measure. To prove that C_φ maps $\mathcal{N}_\alpha$ (continuously) into $\mathcal{N}_\beta$, we follow [18] and [2] and divide Δ into dyadic boxes. Let $\mathcal{D}$ denote the family of dyadic arcs in $\partial\Delta$, that is, the family of all arcs of the form

$$\left\{z \in \partial\Delta : \frac{2\pi k}{2^n} \leqslant \arg z < \frac{2\pi(1+k)}{2^n}\right\}, \quad k = 0,1,\ldots,2^n-1,\ n = 0,1,\ldots.$$

Given any arc I in $\partial\Delta$, let $H(I)$ denote the half of $S(I)$ which is closest to the origin, that is,

$$H(I) = \{z \in S(I) : 1-|I| \leqslant |z| < 1-|I|/2\}.$$

Note that the $H(I)$'s for $I \in \mathcal{D}$ are pairwise disjoint and cover Δ. Fix any enumeration $\{H_j : j = 1,2,\ldots\}$ of these sets and select a point a_j in each H_j. Almost any point would work but in order to simplify some parts later on in the proof let us agree that a_j is the 'center' of H_j in the sense that $|a_j|$ and $\arg a_j$ bisect the interval of absolute values and the interval of arguments, respectively, of points in H_j. If $H_j = H(I)$ then $|I| \asymp 1 - |a_j|$. Given $f \in \mathcal{N}_\alpha$, let $a_j^* \in \overline{H}_j$ (closure of H_j) be a point where $\log(1+|f|)$ attains its maximum on $\overline{H}_j$. If n is such that H_j is contained in $A_n := \{z \in \Delta : 1 - 2^{-n} \leqslant |z| < 1 - 2^{-(n+1)}\}$, then the set $S_j := \{z \in \Delta : 1 - 2^{-(n+1)} \leqslant |z| < 1 - 2^{-(n+2)}, |\arg z - \arg a_j^*| < 2^{-n-1}\}$ contains a disc Δ_j with center a_j^* and radius comparable to 2^{-n}. Note that S_j intersects at most 6 of the sets H_k and that $1 - |z|^2 \asymp 2^{-n}$ whenever $z \in S_j$. Using these observations and the submean value property of $\log(1+|f|)$, we get

$$\begin{aligned}\int_\Delta \log(1+|f|)\,\mathrm{d}m_{\beta,\varphi} &= \sum_j \int_{H_j} \log(1+|f|)\,\mathrm{d}m_{\beta,\varphi} \\ &\leqslant \sum_j \sup_{w\in H_j} \log(1+|f(w)|)\, m_{\beta,\varphi}(H_j) \leqslant C \sum_j \log(1+|f(a_j^*)|)(1-|a_j|^2)^{\alpha+2} \\ &\leqslant C \sum_j \int_{\Delta_j} \log(1+|f(z)|)\,\mathrm{d}m_\alpha(z) \leqslant C \sum_j \int_{H_j} \log(1+|f(z)|)\,\mathrm{d}m_\alpha(z) \\ &\leqslant C \int_\Delta \log(1+|f(z)|)\,\mathrm{d}m_\alpha(z) = C\,|||f|||_\alpha,\end{aligned}$$

as desired.

(ii) is just the 'little o' version of (i) and has a similar proof. Only minor modifications are needed; for example, the test functions in (i) should be replaced by

$$z \mapsto (1-|w|^2)\exp\left[\left(\frac{1-|w|^2}{(1-\overline{w}z)^2}\right)^{\alpha+2}\right], \qquad |w| = 1-h. \quad \blacksquare$$

4. PROOF OF THEOREM 1.2

Suppose first that $C_\varphi : \mathcal{N}_\alpha \to \mathcal{N}_\beta$ is order bounded: for each $s > 0$, there is a $g_s \in L_{\log^+}(m_\beta)$ such that $g_s \geqslant 0$ and $|C_\varphi f(z)| \leqslant g_s(z)$ for all $f \in \mathcal{N}_\alpha$ with $|||f|||_\alpha \leqslant s$ and almost all $z \in \Delta$. Consequently, for almost all $w \in \Delta$, $g_s(w) \geqslant \sup\{|f(\varphi(w))| : f \in \mathcal{N}_\alpha, |||f|||_\alpha \leqslant s\}$. We claim that there is a constant κ, depending only on α and s, such that

$$f_w(z) = \exp\left[\kappa\left(\frac{1-|\varphi(w)|^2}{(1-\overline{\varphi(w)}z)^2}\right)^{\alpha+2}\right] - 1$$

belongs to $B_{\mathcal{N}_\alpha}(0,s)$.

This can be seen through the following modification of an argument from [11]. We start by fixing $s > 0$ and $w \in \Delta$ such that $\varphi(w) \neq 0$ and choose then $\varepsilon > 0$ such that $\log(1+\varepsilon) + (2\varepsilon/\pi) < s$ and $\delta = \delta(\varepsilon) > 0$ such that $|\mathrm{e}^z - 1| < \varepsilon$ whenever $|z| < \delta$. We put $\eta := \delta\left(\dfrac{1-\cos\varepsilon}{2}\right)^{\alpha+2}$.

Consider the sector $S(w,\varepsilon) := \{z \in \Delta : |\arg z - \arg \varphi(w)| < \varepsilon\}$. If $z \in \Delta \setminus S(w,\varepsilon)$, then $|1-\overline{\varphi(w)}z| \geqslant 1-\cos\varepsilon$, hence

$$\left|\frac{1-|\varphi(w)|^2}{(1-\overline{\varphi(w)}z)^2}\right| \leqslant \frac{2}{|1-\overline{\varphi(w)}z|} \leqslant \frac{2}{1-\cos\varepsilon}.$$

This implies that if we choose $\kappa < \eta$, then $|f_w(z)| < \varepsilon$ for $z \in \Delta \setminus S(w,\varepsilon)$, hence

$$\int\limits_{\Delta\setminus S(w,\varepsilon)} \log(1+|f_w|)\,\mathrm{d}m_\beta \leqslant \log(1+\varepsilon).$$

As before, Lemma 4.2.2 from [23] provides a constant K such that

$$\int\limits_{S(w,\varepsilon)} \log(1+|f_w|)\,\mathrm{d}m_\beta \leqslant \int\limits_{S(w,\varepsilon)} (1+\log^+|f_w|)\,\mathrm{d}m_\beta \leqslant 2\,m_\beta(S(w,\varepsilon))+\kappa K = \frac{2\varepsilon}{\pi}+\kappa K.$$

Combining these two estimates, we obtain

$$|||f_w|||_\alpha \leqslant \log(1+\varepsilon) + \frac{2\varepsilon}{\pi} + \kappa K.$$

If we now choose κ such that in addition $\kappa K \leqslant s - \log(1+\varepsilon) - 2\varepsilon/\pi$, then $|||f|||_\alpha \leqslant s$, as asserted.

Using this,

$$g_s(w) \geqslant |f_w(\varphi(w))| = \exp\left[\frac{\kappa}{(1-|\varphi(w)|^2)^{\alpha+2}}\right] - 1,$$

is immediate, and so $(1-|\varphi|^2)^{-(\alpha+2)} \in L^1(m_\beta)$.

Conversely, if φ is such that $(1-|\varphi|^2)^{-(\alpha+2)} \in L^1(m_\beta)$ then (2.2) yields

$$\log(1+|C_\varphi f(z)|) \leqslant \frac{M_0|||f|||_\alpha}{(1-|\varphi(z)|^2)^{\alpha+2}}. \tag{4.1}$$

Thus $C_\varphi f \in \mathcal{N}_\beta$ and $C_\varphi : \mathcal{N}_\alpha \to \mathcal{N}_\beta$ is order bounded, 'order bounds' being the functions $z \mapsto \exp(M_0\,s\,(1-|\varphi(z)|^2)^{-(\alpha+2)})$, $s > 0$. ∎

5. PROOF OF THEOREM 1.4

Without loss of generality, we may assume that $\varphi(0) = 0$; otherwise we can replace φ with $\tau \circ \varphi$ where τ is the Möbius transform of Δ which exchanges 0 and $\varphi(0)$.

First of all, if f is in $\mathcal{A}^p_\alpha$ with $\|f\|_{p,\alpha} \leqslant 1$ then, as in [21], we get

$$|C_\varphi f(z)| \leqslant \frac{1}{(1-|\varphi(z)|^2)^{(\alpha+2)/p}} \tag{5.1}$$

for all $z \in \Delta$. By Schwarz's lemma, $|\varphi(z)| \leqslant |z|$ for all $z \in \Delta$, hence

$$\begin{aligned} |||C_\varphi f|||_\beta &\leqslant \int\limits_\Delta \log\left(1+\frac{1}{(1-|\varphi(z)|^2)^{(\alpha+2)/p}}\right)\mathrm{d}m_\beta(z) \\ &\leqslant \int\limits_\Delta \log\left(1+\frac{1}{(1-|z|^2)^{(\alpha+2)/p}}\right)\mathrm{d}m_\beta(z) \leqslant C. \end{aligned}$$

It follows that, regardless of which analytic function $\varphi : \Delta \to \Delta$ we choose, C_φ exists as a continuous operator $\mathcal{A}^p_\alpha \to \mathcal{N}_\beta$.

Next we pass to compactness. Let (f_n) be a sequence in the unit ball of $\mathcal{A}^p_\alpha$. Linearity allows us, passing to a subsequence if necessary, to assume that (f_n) converges l.u. to 0. We must prove that $\lim_n |||C_\varphi f_n|||_\beta = 0$. Given $0 < \delta < 1$, write $|||C_\varphi f_n|||_\beta = I_n + J_n$ where

$$I_n := \int\limits_{|z|\leqslant\delta} \log\left(1 + |f_n \circ \varphi|\right) \mathrm{d}m_\beta \quad \text{and} \quad J_n := \int\limits_{|z|>\delta} \log\left(1 + |f_n \circ \varphi|\right) \mathrm{d}m_\beta.$$

By compactness of $\{|z| \leqslant \delta\}$, (I_n) is a null sequence. In order to deal with the J_n's, we apply (5.1). Accordingly,

$$J_n = \int\limits_{|z|>\delta} \log\left(1 + |f_n \circ \varphi)|\right) \mathrm{d}m_\beta \leqslant \int\limits_{|z|>\delta} \log\left(1 + \frac{1}{(1-|\varphi(z)|^2)^{(\alpha+2)/p}}\right) \mathrm{d}m_\beta(z)$$
$$\leqslant \int\limits_{|z|>\delta} \log\left(1 + \frac{1}{(1-|z|^2)^{(\alpha+2)/p}}\right) \mathrm{d}m_\beta(z).$$

Since the last integral tends to 0 as $\delta \to 1$, we are done by Proposition 2.3.

To see that $C_\varphi : \mathcal{A}^p_\alpha \to \mathcal{N}_\beta$ is actually order bounded, look at the functions

$$f_w(z) = \left(\frac{1-|\varphi(w)|^2}{(1-\overline{\varphi(w)}z)^2}\right)^{(\alpha+2)/p}.$$

Since $\int\limits_\Delta \log \frac{1}{1-|z|^2} \,\mathrm{d}m_\beta(z) < \infty$, our claim follows from (5.1). ∎

6. PROOF OF THEOREM 1.5

Suppose that $C_\varphi : \mathcal{N}_\alpha \to \mathcal{A}^q_\beta$ exists as a continuous operator. Suppose we are given $\theta \in [0, 2\pi)$, $h \in (0,1)$ sufficiently small, and $M > 0$. Write $w = (1-h)\mathrm{e}^{\mathrm{i}\theta}$. This time we choose

$$g_w(z) = \exp\left[\frac{M}{q} \cdot \left(\frac{1-|w|^2}{(1-\overline{w}z)^2}\right)^{\alpha+2}\right]$$

as our test function. It is easy to see that $g_w \in \mathcal{N}_\alpha$ with $\|g_w\|_{\mathcal{N}_\alpha} \leqslant C$ where C depends on M, q and α. Let again I be the arc with center at $\mathrm{e}^{\mathrm{i}\theta}$ and $|I| = h$. For $z \in S(I)$ we get, again since $1/(|1-\overline{w}z|)^{4(\alpha+2)} \geqslant c/h^{4(\alpha+2)}$ and $\mathrm{Re}(1-w\overline{z})^{2(\alpha+2)} \geqslant c\, h^{2(\alpha+2)}$,

$$\|C_\varphi g_w\|_{q,\beta} \geqslant \int\limits_{S(I)} |g_w(z)|^q \mathrm{d}m_{\beta,\varphi}(z) \geqslant \exp\left(\frac{c^2 M}{h^{\alpha+2}}\right) \cdot m_{\beta,\varphi}\big(S(I)\big).$$

This yields

$$m_{\beta,\varphi}(S(I)) = \mathrm{O}\left[\exp\left(-\frac{M}{|I|^{\alpha+2}}\right)\right] \tag{6.1}$$

for all $M > 0$.

We claim that (6.1) implies that for each $K > 0$

$$\exp\left[\frac{K}{(1-|\varphi|^2)^{\alpha+2}}\right] \in L^1(m_\beta). \tag{6.2}$$

For this we return to the decomposition of Δ used in the proof of part (i) of Theorem 1.1. If $H_j = H(I)$ then $c_1(1-|a_j|^2)^{-(\alpha+2)} \leqslant |I|^{-(\alpha+2)} \leqslant C_1(1-|a_j|^2)^{-(\alpha+2)}$, and if $z \in \Delta_j$ then $c_2(1-|a_j|^2)^{-(\alpha+2)} \leqslant (1-|z|^2)^{-(\alpha+2)} \leqslant C_2\,(1-|a_j|^2)^{-(\alpha+2)}$. Let $z_j \in \Delta_j$ be such that $(1-|z_j|^2)^{-(\alpha+2)} = \max_{z\in\overline{H}_j}(1-|z|^2)^{-(\alpha+2)}$. Then $c_3(1-|a_j|^2)^{-(\alpha+2)} \leqslant (1-|z_j|^2)^{-(\alpha+2)} \leqslant C_3(1-|a_j|^2)^{-(\alpha+2)}$. Here the c_j's and C_j's are absolute constants. Fix $K > 0$ and choose $M > K\,C_3/c_1$. We get from (6.1)

$$\begin{aligned}
&\int_\Delta \exp\big((K(1-|z|^2)^{-(\alpha+2)}\big)\,\mathrm{d}m_{\beta,\varphi}(z)\\
&\qquad= \sum_j \int_{H_j} \exp\big(K(1-|z|^2)^{-(\alpha+2)}\big)\,\mathrm{d}m_{\beta,\varphi}(z)\\
&\qquad\leqslant \sum_j \sup_{z\in H_j} \exp\big(K(1-|z|^2)^{-(\alpha+2)}\big)\,m_{\beta,\varphi}(H_j)\\
&\qquad\leqslant C\sum_j \exp(K(1-|z_j|^2)^{-(\alpha+2)}\big)\exp\big(-Mc_1(1-|a_j|^2)^{-(\alpha+2)}\big)\\
&\qquad\leqslant C\sum_j \exp\big((KC_3-Mc_1)(1-|a_j|^2)^{-(\alpha+2)}\big)\\
&\qquad\leqslant C\sum_j \int_{\Delta_j} \exp\left(\frac{KC_3-Mc_1}{C_2(1-|z|^2)^{\alpha+2}}\right)\frac{\mathrm{d}m(z)}{(1-|z|^2)^2}\\
&\qquad\leqslant C\sum_j \int_{H_j} \exp\left(\frac{KC_3-Mc_1}{C_2(1-|z|^2)^{\alpha+2}}\right)\frac{\mathrm{d}m(z)}{(1-|z|^2)^2}\\
&\qquad\leqslant C\int_\Delta \exp\left(\frac{KC_3-Mc_1}{C_2(1-|z|^2)^{\alpha+2}}\right)\frac{\mathrm{d}m(z)}{(1-|z|^2)^2} \leqslant \frac{CC_2}{Mc_1-KC_3}\,m_\alpha(\Delta),
\end{aligned}$$

which gives (6.2). Conversely, it is readily seen that (6.2) implies that $C_\varphi : \mathcal{N}_\alpha \to \mathcal{A}^q_\beta$ is order bounded. Of course, such an operator is always continuous, so that only compactness requires an argument.

But this is easy. The condition in the theorem is independent of q, so that, for each $r > q$, $C_\varphi : \mathcal{N}_\alpha \to \mathcal{A}^q_\beta$ admits a factorization $\mathcal{N}_\alpha \to \mathcal{A}^r_\beta \hookrightarrow \mathcal{A}^q_\beta$. By atomic decomposition, $\mathcal{A}^r_\beta$ is isomorphic to ℓ^r and $\mathcal{A}^q_\beta$ is isomorphic to ℓ^q; compare [5] and [17] for a proof. So our claim follows from Pitt's theorem; see for example [16], p. 208. ∎

Acknowledgements. Much of this research has been done while the second named author was visiting the University of Zürich in October 1998. He would like to thank for support and excellent working conditions.

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Received May 27, 1999