J. OPERATOR THEORY
53:2(2005), 441–442

CORRECTION TO THE PAPER "REAL STRUCTURE IN PURELY INFINITE C^*-ALGEBRAS"

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Communicated by Kenneth R. Davidson

ABSTRACT. In order to correct an error in the paper: P.J. Stacey, Real structure in purely infinite C^*-algebras, *J. Operator Theory* **49**(2003), 77–84, a Kishimoto type result for involutory antiautomorphisms is proved.

KEYWORDS: *C^*-algebra, purely infinite, involutory $*$-antiautomorphism.*

MSC (2000): Primary 46L05; Secondary 46L35.

The first author has discovered an error in Lemma 3.1 of [5]. A counterexample to that claim is obtained by taking A to be commutative and Φ to be the identity. The error arises by not using the correct relationship $(Vk, h) = \overline{(k, V^*h)}$ between an antilinear map on a Hilbert space and its adjoint.

The following proposition can be substituted for the incorrect lemma to leave the remaining results of the paper intact. A similar result has been obtained in [2].

PROPOSITION 1. *Let A be a non-commutative C^*-algebra and let Φ be an involutory $*$-antiautomorphism of A. Then there exists a non-zero positive element $x \in A$ with $x\Phi(x) = 0$.*

Proof. Assume firstly that A contains a non-zero element b with $\Phi(b) = b^*$ and $b^2 = 0$. Then let $y = \mathrm{i}b - \mathrm{i}b^* + (bb^*)^{1/2} + (b^*b)^{1/2}$. If $y = 0$ then, premultiplying and postmultiplying by b and noting that $b(bb^*)^{1/2} = 0$ and $(b^*b)^{1/2}b = 0$, it follows that $bb^*b = 0$ and thus that $b = 0$. Therefore $y \neq 0$. For each polynomial p on the spectrum of bb^* with $p(0) = 0$, $b^*p(bb^*) = p(b^*b)b^*$ and therefore $b^*(bb^*)^{1/2} = (b^*b)^{1/2}b^*$ and $b(b^*b)^{1/2} = (bb^*)^{1/2}b$. It follows that the self-adjoint element y satisfies $y\Phi(y) = 0$ and then that $x = y^2$ has the required properties.

If the real algebra $R = \{a \in A : \Phi(a) = a^*\}$ does not contain a non-zero element b with $b^2 = 0$, then neither does the algebra $\widetilde{R}$ obtained by adjoining a unit. It will be shown that every irreducible representation of $\widetilde{R}$ (and hence of R) has image $\mathbb{R}, \mathbb{C}$ or $\mathbb{H}$, the algebra of quaternions. Note firstly that the argument in Exercise 4.6.30 of [3] applies directly to real algebras to show that every closed left ideal of R is 2-sided. For any pure real state k of $\widetilde{R}$, it is shown in Proposition

5.3.8 of [4] that $I_k = \{a \in \widetilde{R} : k(a^*a) = 0\}$ is a maximal left ideal of $\widetilde{R}$ and is closed. It is therefore 2-sided, from which it follows that the kernel of the associated irreducible representation π_k of $\widetilde{R}$ is equal to $\{b \in \widetilde{R} : ba \in I_k \text{ for all } a \in \widetilde{R}\} = I_k$. If y is a non-invertible, non-zero element of $\pi_k(\widetilde{R})$, then it generates either a proper left ideal or a proper right ideal of $\pi_k(\widetilde{R})$, for which either the inverse image or its adjoint is a proper left ideal of $\widetilde{R}$ strictly containing I_k, which yields a contradiction. Thus $\pi_k(\widetilde{R})$ is a real Banach division algebra and therefore is isomorphic to $\mathbb{R}, \mathbb{C}$ or $\mathbb{H}$.

If π is an irreducible representation of A then, since every irreducible representation of R has image $\mathbb{R}, \mathbb{C}$ or $\mathbb{H}$, a standard argument shows that $\pi(A)$ is either isomorphic to $\mathbb{C}$ or $M_2(\mathbb{C})$. If the image is $M_2(\mathbb{C})$ then, because $\pi(R)$ is isomorphic to $\mathbb{H}$ rather than $M_2(\mathbb{C})$, the kernel of π is fixed by Φ. The intersection J of the kernels of all 1-dimensional representations is also invariant under Φ and is a 2-homogeneous algebra which is non-zero by the hypothesis that A is non-commutative. From Theorem 3.2 of [1], J has a non-zero subalgebra of the form $C_0(Y, M_2(\mathbb{C}))$ for some locally compact Hausdorff subspace Y of $\mathrm{Prim}(A)$. Φ acts trivially on Y and therefore, for each $f \in C_0(Y, M_2(\mathbb{C}))$ and each $y \in Y$, $(\Phi f)(y) = \Phi_y(f(y))$ for some involutory antiautomorphism Φ_y of $M_2(\mathbb{C})$ with associated real algebra isomorphic to $\mathbb{H}$. There is a unique such involutory antiautomorphism, given by $\Phi_y\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. The element $x = \begin{pmatrix} f & 0 \\ 0 & 0 \end{pmatrix}$, where f is a non-zero positive function, therefore has the required properties. ∎

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Received February 28, 2005.